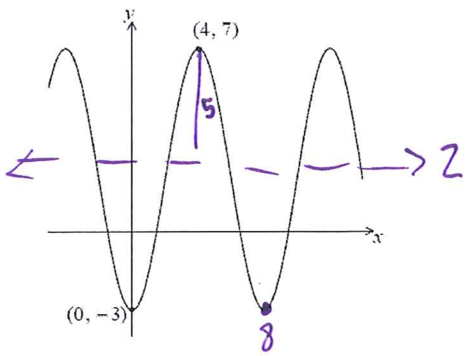
SL 1 Test 3.1 Review Day 3

Foundational (6 points each):

ONE:

The graph of $y = p\cos qx + r$, for $-5 \le x \le 14$, is shown below.



$qp = 2\pi$

$q8 = 2\pi$

$q = \frac{\pi}{4}$

There is a minimum point at $(0, -3)$ and a maximum point at $(4, 7)$.

(a) Find the value of

(i) p; 5

(ii) q; $\pi/4$

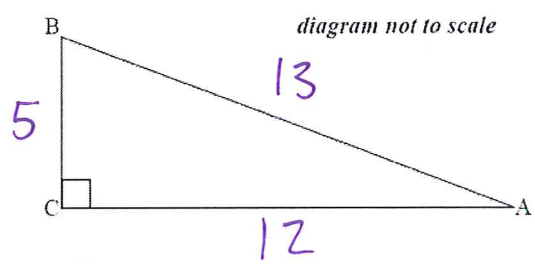
(iii) r. 2 *[6 marks]*

(b) The equation $y = k$ has exactly **two** solutions. Write down the value of k. $k=-3$ *[1 mark]*

TWO:

The triangle is not currently drawn on the x, y plane. If it was, it would be in quadrant 1 with angle A at the origin.

The following diagram shows a right-angled triangle, ABC, where $\sin A = \frac{5}{13}$.



$\sqrt{13^2-5^2} = \sqrt{144} = 12$

(a) Show that $\cos A = \frac{12}{13}$.

(b) Find $\cos 2A$.

$2\left(\frac{12}{13}\right)^2 - 1$

$2\left(\frac{144}{169}\right) - \frac{169}{169}$

$= \frac{288-169}{169}$

Moderate (4 points each):

THREE:

Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

(a) Find $f\left(\frac{\pi}{2}\right)$. $\longrightarrow \cos(\pi) = -1$ [2 marks]

(b) Find $(g \circ f)\left(\frac{\pi}{2}\right)$. $2(-1)^2 - 1 = 2 - 1 = 1$ [2 marks]

(c) Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of k, $k \in \mathbb{Z}$. [3 marks]

$2\cos^2(2x) - 1 = \cos(4x)$

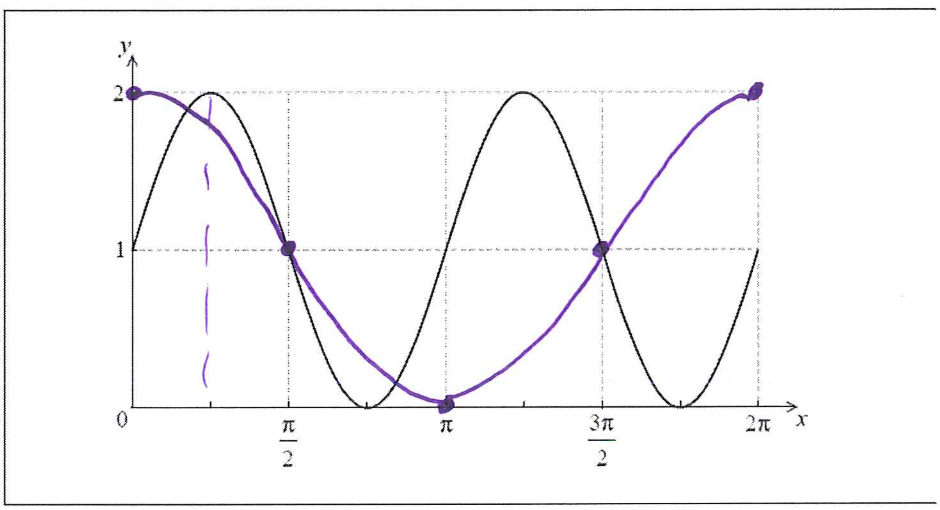
FOUR:

Let $f(x) = (\sin x + \cos x)^2$.

(a) Show that $f(x)$ can be expressed as $1 + \sin 2x$.

$\sin^2 x + 2\sin x \cos x + \cos^2 x$
$= 1 + \sin 2x$

The graph of f is shown below for $0 \le x \le 2\pi$.



$p = 2$

$k = \frac{\pi}{4}$

(b) Let $g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of g for $0 \le x \le 2\pi$.

The graph of g can be obtained from the graph of f under a horizontal stretch of scale factor p followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$.

FIVE:

Consider $y = \sin\left(x + \frac{\pi}{9}\right)$.

$0 = \sin(a)$ $\quad a = 0, \pi, 2\pi$

$x = \cancel{-\frac{\pi}{9}}, \frac{8\pi}{9}, \cancel{\frac{17\pi}{9}}$

(a) The graph of y intersects the x-axis at point A. Find the x-coordinate of A, where $0 \le x \le \pi$.

(b) Solve the equation $\sin\left(x + \frac{\pi}{9}\right) = -\frac{1}{2}$, for $0 \le x \le 2\pi$.

$a = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\frac{33\pi}{18} - \frac{2\pi}{18} = \frac{31\pi}{18}$

$x = \frac{7\pi}{6} - \frac{\pi}{9} = \frac{21\pi}{18} - \frac{2\pi}{18} = \frac{19\pi}{18}$