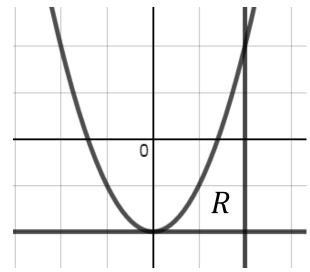
6.3 HW AP Calculus AB **Name:**________________________________

Volumes of Revolution: Disks and Washers

1. A region enclosed by the x-axis, $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$, $and\ y = \cos x$. Write an expression that represents the volume of the solid when the region is rotated about the x-axis.

2. Find the volume of the solid generated when the region R enclosed by the line $y = -2$, the line $x = 2$ and the curve $f(x) = x^2 - 2$, when revolved about the line $y = -2$.



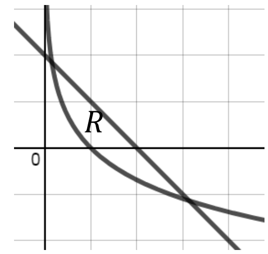
3. What is the volume of the solid generated when the region enclosed by $y = x^2$, the $y - axis$, and $y = 1$ is rotated about the line $= 1$?

4. Find the volume of the solid generated when the region enclosed by $y = x^3$, $y = 1$, the line $x = 3$, and the x axis, is rotated about the line $y = 1$.

5. Write an expression representing the volume of the solid generated when the region enclosed by $y = \ln(x+1)$, the line $= 4$, and the x-axis when rotated about the x-axis.

6. Find the volume of the solid generated when the region enclosed by $y = 2\sin x$, the y-axis, and $y = 2$, is rotated about $y = 2$.

7. **(Calculator allowed)**. Let R be the region bounded by the graph of $y = -x + 2$ and $y = -\ln(x)$ as shown.



a) Find the volume of the solid generated when R is rotated about a horizontal line $y = -2$.

b) Find the volume of the solid generated when R is rotated about the y-axis.

8. Let R be the region in the first quadrant bounded by the graph of $y = \sqrt{x}$ and $= \frac{x}{2}$. Find the volume of the solid generated when R is rotated about the vertical line $x = -2$.

9. **(Calculator allowed)**. Let R be the region enclosed by the graphs of $y = e^{-x}$ and $y = (x-1)^2$.

a) Find the volume of the solid generated when R is revolved about the x-axis.

b) Set up, but do not evaluate, an integral expression for the volume of the solid generated when R is revolved about the y-axis.

10. Let R be the region bounded by the x-axis, the y-axis, the graph of $y = \sqrt{x} + 1$ and the line $x = 4$.

a) Find the volume of the solid generated when R is revolved about the x-axis.

b) Find the volume of the solid when R is revolved about the y-axis.

11. Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x-1}$ and the line $= 5$. The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis they generate solids of equal volumes. Find the value of k.

12. The region bounded by $y = e^{-x}$, $y = 1$, and $x = 3$ is rotated about the x-axis. Find the volume of the solid generated.

13. A region is bounded by $y = 1 + x^2$, and $y = 5$. Find the volume of the solid generated when the region is rotated:

a) About the x-axis

b) About the y-axis

c) About $y = -1$

d) About $x = -2$

14. A solid generated when the region in the first quadrant enclosed by $y = (x^2 - 1)^2$, the x-axis, and the y-axis, is revolved about the x-axis. The volume is found by evaluating which of the following integrals?

$A. \pi \int_0^1 (x^2 - 1)^2 dx$ $\quad B.\ \pi \int_0^9 (x^2 - 1)^2 dx$ $\quad C.\ 2\pi \int_0^1 (x^2 - 1)^4\, dx$

$D.\ \pi \int_0^1 (x^2 - 1)^4\, dx$ $\quad E. \pi \int_{-1}^1 (x^2 - 1)^4 dx$

15. Find the volume of a solid generated by revolving about the y axis the region enclosed by the graphs of $y = 4 - x^2$ and $y = 4 - 2x$.

ANSWERS – Volumes of Revolution: Disks and Washers

1. $\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos x)^2 \, dx$
2. $\frac{32\pi}{5}$
3. $\frac{8\pi}{15}$
4. $\frac{33\pi}{14}$
5. $\pi \int_0^4 (\ln(x+1))^2$
6. $3\pi^2 - 8\pi$
7. a) 27.033

   b) 17.099
8. $\frac{48\pi}{5}$
9. a) 0.845

   b) $\pi \int_0^{.228} [(1+\sqrt{y})^2 \text{-} (1-\sqrt{y})^2] dy + \pi \int_{.228}^{1} [(-\ln y)^2 - \left(1-\sqrt{y}\right)^2] dy$
10. a) $\frac{68\pi}{3}$

    b) $\frac{208\pi}{5}$
11. $1 + 2\sqrt{2}$
12. $\pi[\frac{5}{2} + \frac{1}{2e^6}]$
13. a) $\frac{1088\pi}{15}$

    b) 8π

    c) $\frac{1408\pi}{15}$

    d) $\frac{128\pi}{3}$
14. D
15. $\frac{8\pi}{3}$