$$A = \begin{bmatrix} 2 & -6 \\ -5 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A \cdot B$$

$$B \cdot A$$

THE IDENTITY MATRIX:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2 x 2 identity

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3 x 3 identity

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The 4 x 4 identity

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*All identity matrices are square.

It's ones down this **main diagonal**
and zeros everywhere else.

Multiplying a number by 1 gives us back the same number:

$$3 \cdot 1 = 3 \quad \text{or} \quad 1 \cdot a = a$$

The identity matrix does the same thing for matrices:

$$A = \begin{bmatrix} 2 & -6 \\ -5 & 8 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A \cdot I = A \quad \text{AND} \quad I \cdot A = A$$

We've now learned to add and subtract matrices... how to multiply a matrix by a scalar... and how to multiply two matrices...

So, what about division?

Well, there isn't a division process for matrices. BUT, there IS a way to get around this little problem. We can use something we already know about: multiplication.

If we solve: $3x = 4$

Let's take a closer look at what's going on here:

Instead of saying that we are dividing by 3, how about saying that we are multiplying by $\frac{1}{3}$:

$$3x = 4$$

$$\frac{1}{3} \cdot 3x = \frac{1}{3} \cdot 4$$

$$x = \frac{4}{3}$$

The result is the same, but we used multiplication instead of division.

Let's stick letters in for the numbers:

So, if we have the inverse of a matrix, we can "divide"

$$\begin{bmatrix} a & b \end{bmatrix}^{-1} \qquad 1 \qquad \begin{bmatrix} d & -b \end{bmatrix}$$

If we have a matrix

$$B = \begin{bmatrix} 2 & 3 \\ -4 & -5 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 3 \\ -4 & -5 \end{bmatrix} \qquad A = \begin{bmatrix} 3 & 8 \\ -6 & 5 \end{bmatrix}$$

Solve for x:

$$[B]x = [A]$$

Because we can't multiply,

$$x = [B]^{-1}[A]$$

$$B = \begin{bmatrix} 2 & 3 \\ -4 & -5 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$X = \begin{array}{cc} [B]^{-1} & [A] \\ \begin{bmatrix} -2.5 & -1.5 \\ 2 & 1 \end{bmatrix} & \begin{bmatrix} 3 & 8 \\ -6 & 5 \end{bmatrix} \end{array}$$

Find the inverse of this matrix, then check it:

$$A = \begin{bmatrix} 3 & 8 \\ -6 & 5 \end{bmatrix}$$

So, what's all this fuss about inverse matrices?

You can use them to solve systems of equations.

Check it out:

$$2x - 5y = 8$$

$$3x + 4y = -6$$

Grab the coefficients and make a matrix:

$$A = \begin{bmatrix} 2 & -5 \\ 3 & 4 \end{bmatrix}$$

$$\underline{A} = \begin{bmatrix} 2 & -5 \\ 3 & 4 \end{bmatrix} \qquad \begin{aligned} 2x - 5y &= 8 \\ 3x + 4y &= -6 \end{aligned}$$

Then, we'll need matrices for

$$\underline{X} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} 8 \\ -6 \end{bmatrix}$$

Our variables **Our answers**

In short, this system can be rewritten as

$$\begin{bmatrix}2 & -5\\ 3 & 4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}8\\ -6\end{bmatrix}$$

$$AX = B$$

and we can use algebra to solve it!

$$AX = B$$

$$A^{-1}\cdot AX = A^{-1}\cdot B$$

$$X = \underline{A^{-1}\cdot B}$$

order REALLY matters here!

$$\begin{bmatrix} \underset{a}{2} & \underset{b}{4} \end{bmatrix}$$

$$A^{-1} = \frac{1}{2\cdot 4 - -5\cdot 3}\begin{bmatrix} 4 & 5 \\ -3 & 2 \end{bmatrix} = \frac{1}{23}\begin{bmatrix} 4 & 5 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 4/23 & 5/23 \\ -3/23 & 2/23 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix}^{-1} \quad 1 \quad \begin{bmatrix} d & -b \end{bmatrix}$$

So, let's find $A^{-1} \bullet B$:

$$\begin{bmatrix} 8 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} \frac{4}{23} & \frac{5}{23} \\ \frac{-3}{23} & \frac{2}{23} \end{bmatrix} \begin{bmatrix} 2/23 \\ -36/23 \end{bmatrix} \leftarrow \text{This is } X$$

$$\frac{4}{23} \cdot 8 + \frac{5}{23} \cdot {}^{-}6$$

$$\frac{32}{23} + \frac{-30}{23} = 2/23$$

$$-3/23 \cdot 8 + \frac{2}{23} \cdot {}^{-}6$$

$$\frac{-24}{23} + \frac{-12}{23}$$

$$= {}^{-}36/23$$

$$\begin{cases} 2x - 5y = 8 \\ 3x + 4y = -6 \end{cases}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{2}{23} \\ \frac{-36}{23} \end{bmatrix}$$

So, $x = \frac{2}{23}$ and $y = \frac{-36}{23}$

You try:

Use inverse matrices to solve:

$$3x - 6y = 5$$

$$-4x + 8y = -1$$

Use inverse matrices to solve:

$$8x - 7y = 4$$

$$2x + 9y = -6$$

How to multiply matrices using a calculator

$$A = \begin{bmatrix} -4 & 0 \\ 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 1 \\ -4 & 0 \end{bmatrix}$$