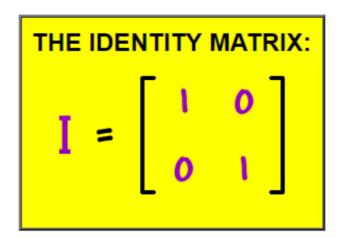
$$A = \begin{bmatrix} 2 & -6 \\ -5 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A \cdot B$$

$$B \cdot A$$



## 2 x 2 identity

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The 4 x 4 identity

## 3 x 3 identity

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

\*All identity matrices are square.

It's ones down this main diagonal and zeros everywhere else.

Multiplying a number by 1 gives us back the same number:

$$3 \cdot 1 = 3$$
 or  $1 \cdot a = a$ 

The identity matrix does the same thing for matrices:

$$A = \begin{bmatrix} 2 & -6 \\ -5 & 8 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A \cdot I = A \quad AND \quad I \cdot A = A$$

We've now learned to add and subtract matrices... how to multiply a matrix by a scalar... and how to multiply two matrices...

So, what about division?

Well, there isn't a division process for matrices. BUT, there IS a way to get around this little problem. We can use something we already know about: multiplication.

If we solve: 3x = 4

Let's take a closer look at what's going on here:

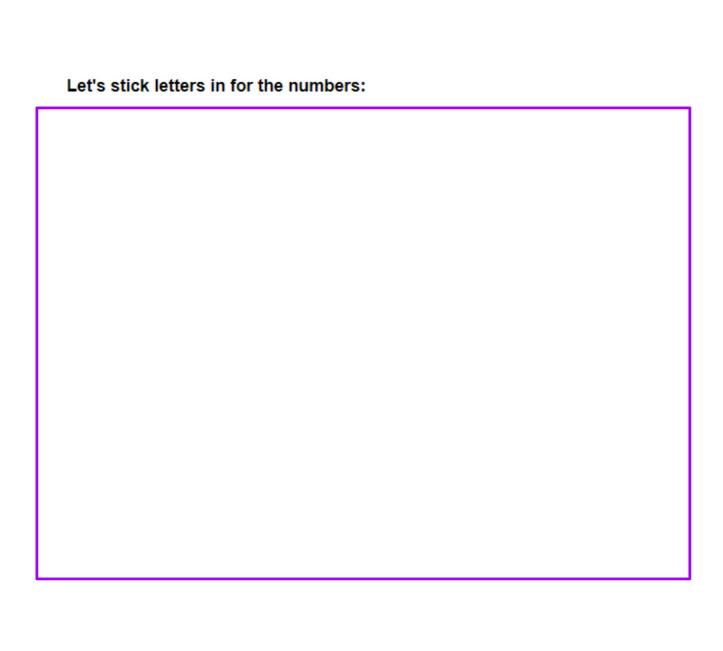
Instead of saying that we are dividing by  $\frac{3}{3}$ , how about saying that we are multiplying by  $\frac{1}{3}$ :

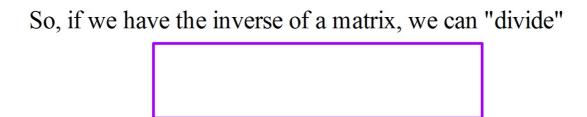
$$3X = 4$$

$$\frac{1}{3} \cdot 3X = \frac{1}{3} \cdot 4$$

$$X = \frac{4}{3}$$

The result is the same, but we used multiplication instead of division.





$$\lceil a \ b \rceil^{-1}$$
 1  $\lceil d \ -b \rceil$ 

If we have a matrix

$$B = \begin{bmatrix} 2 & 3 \\ -4 & -5 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 3 \\ -4 & -5 \end{bmatrix} \qquad A = \begin{bmatrix} 3 & 8 \\ -6 & 5 \end{bmatrix}$$

Solve for x:

$$[B]x = [A]$$

Because we can't multiply,

$$x = [B]^{-1}[A]$$

$$B = \begin{bmatrix} 2 & 3 \\ -4 & -5 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$X = \begin{bmatrix} -2.5 & -1.5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 8 \\ -6 & 5 \end{bmatrix}$$

Find the inverse of this matrix, then check it:

$$A = \begin{bmatrix} 3 & 8 \\ -6 & 5 \end{bmatrix}$$

So, what's all this fuss about inverse matrices?

You can use them to solve systems of equations.

Check it out:

$$2x - 5y = 8$$

$$3x + 4y = -6$$

Grab the coefficients and make a matrix:

$$A = \begin{bmatrix} 2 & -5 \\ 3 & 4 \end{bmatrix}$$

Then, we'll need matrices for

$$\underline{X} = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ -6 \end{bmatrix}$$
Our variables
Our answers

In short, this system can be rewritten as 
$$AX = B$$

$$\begin{bmatrix} 2 & -5 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 9 \\ -6 \\ 3 \end{bmatrix}$$

and we can use algebra to solve it!

$$AX = B$$

$$A^{-1} \cdot AX = A^{-1} \cdot B$$

$$X = A^{-1} \cdot B \leftarrow \text{order REALLY matters here!}$$

$$A = \frac{1}{2.4 - 5.3} \begin{bmatrix} 4 & 5 \\ -3 & 2 \end{bmatrix} = \frac{1}{23} \begin{bmatrix} 4 & 5 \\ -3 & 2 \end{bmatrix}$$

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$$\lceil a \mid b \rceil^{-1}$$
 1  $\lceil d \mid b \rceil$ 

$$2x - 5y = 8$$

$$3x + 4y = -6$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{2}{23} \\ -\frac{36}{23} \end{bmatrix}$$

So, 
$$\frac{x}{23} = \frac{2}{23}$$
 and  $\frac{y}{23} = \frac{-36}{23}$ 

## You try:

Use inverse matrices to solve:

$$3x - 6y = 5$$
  
 $-4x + 8y = -1$ 

Use inverse matrices to solve:

$$8x - 7y = 4$$

$$2x + 9y = -6$$

How to multiply matrices using a calculator

$$A = \begin{bmatrix} -4 & 0 \\ 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 1 \\ -4 & 0 \end{bmatrix}$$