Matrices

A matrix is just a rectangular grid of numbers.

Here are some examples:

$$\begin{bmatrix} 1 & 3 & 0 \\ -2 & 6 & 4 \end{bmatrix} \quad \begin{bmatrix} 13 \\ 2 \\ -4 \end{bmatrix} \quad \begin{bmatrix} 0 & -3 \\ 2 & 10 \end{bmatrix}$$

"Matrices" is the plural of "matrix."

We'll need some terminology...

$$\begin{bmatrix} 1 & 3 & 0 \\ -2 & 6 & 4 \end{bmatrix}$$

These are the entries.

These are the rows

$$\begin{array}{c|c} \begin{bmatrix} 1 & 3 & 0 \\ -2 & 6 & 4 \end{bmatrix} & \begin{matrix} \leftarrow \text{row } 1 \\ \leftarrow \text{row } 2 \end{matrix} \end{array}$$

These are the columns

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \hline \begin{bmatrix} 1 & 3 & 0 \\ -2 & 6 & 4 \end{bmatrix} \\ \hline \uparrow & \uparrow & \uparrow \\ \text{column } 1 & \text{column } 2 & \text{column } 3 \end{array}$$

$$\begin{bmatrix} 1 & 3 & 0 \\ -2 & 6 & 4 \end{bmatrix}$$

has **2** rows and **3** columns.

We call this a 2x3 matrix

Sometimes, we'll need to refer to a specific entry, so we have a special "tagging" system. It's based on rows and columns:

$a_{32} = 3$

a_{ij}

3rd row 2nd c.

a_{ij} is the entry in the i^{th} row and the j^{th} column.

For example:

a_{12} 7

$$\begin{bmatrix} 5 & 4 \\ -2 & 11 \\ 0 & 3 \end{bmatrix}$$

5 is entry a_{11}

-2 is entry a_{21}

0 is entry a_{31}

Adding and subtracting matrices is really easy but, you can onlv do it if thev are the same size!

$$\begin{bmatrix} 1 & 3 & 4 \\ -7 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 6 & -3 & 11 \\ 0 & 5 & -4 \end{bmatrix}$$

We just add the entries in each spot...

$$\begin{bmatrix} 1+6 & 3+(-3) & 4+11 \\ -7+0 & 0+5 & 5+(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 0 & 15 \\ -7 & 5 & 1 \end{bmatrix}$$

Scalar Multiplication

First, it would be nice to know what a scalar is!

Don't worry -- it's easy...

A scalar is just a number like 3 or -5 or $\frac{2}{7}$ or .4

Suppose we have:

$$A = \begin{bmatrix} -4 & 0 \\ 3 & 6 \end{bmatrix}$$

and we need to find 2A... (2 times A)

$$2A = 2\begin{bmatrix} -4 & 0 \\ 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2(-4) & 2(0) \\ 2(3) & 2(6) \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 0 \\ 6 & 12 \end{bmatrix}$$

You just multiply each entry by 2.

$$A = \begin{bmatrix} -4 & 0 \\ 3 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 1 \\ -4 & 0 \end{bmatrix}$$

Find $3B - A$

$$= \begin{bmatrix} 25 & 3 \\ -15 & -6 \end{bmatrix}$$

Multiplying Matrices

When multiplying, the size of the matrix really matters:

The number of columns in the first matrix must equal the number of rows in the second matrix if you want to multiply them

If the size of matrix A is $m \times n$
and the size of matrix B is $n \times p$,

then

These must match.

$$m \times n \cdot n \times p$$

The answer is size $m \times p$.

$$A = \begin{bmatrix} -1 & 5 \\ 2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 9 \\ 8 & -6 \\ 4 & 7 \end{bmatrix}$$

Let's find $B \cdot A$:

$B \cdot A$

$$A \rightarrow \begin{bmatrix} -1 & 5 \\ 2 & -3 \end{bmatrix}$$

$$B \rightarrow \begin{bmatrix} 0 & 9 \\ 8 & -6 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 18 & -27 \\ -20 & 58 \\ 10 & -1 \end{bmatrix} \leftarrow$$

The answer goes here.

$8 \cdot 5 + {-6} \cdot {-3}$

$40 + 18$

$= 58$

$-1 \cdot 4 + 2 \cdot 7$

$-4 + 14$

$= 10$

YOUR TURN:

$$A = \begin{bmatrix} 11 & 6 \\ -3 & -7 \end{bmatrix} \quad B = \begin{bmatrix} -5 & 4 \\ 1 & -8 \end{bmatrix}$$

Find $A \cdot B$ **AND** $B \cdot A$

$$A \cdot B = \begin{bmatrix} -49 & -4 \\ 8 & 44 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} -67 & -58 \\ 35 & 62 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 6 \\ -4 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 8 & 3 \\ -2 & -7 \end{bmatrix}$$

$$C = \begin{bmatrix} 10 & 1 \\ -5 & 0 \end{bmatrix}$$

Find $2A + BC + 3BA$.

***hint: Do the matrix multiplications first, then put everything together slowly.**

$$2A + BC + 3BA = \begin{bmatrix} 55 & 245 \\ 85 & -209 \end{bmatrix}$$