HW14: Unit 1 (Limits) Review

1) Complete the table for the function and find the indicate limit

1) $\lim_{x\to 2}\dfrac{3-x}{x-2}$

x						
$f(x)$						

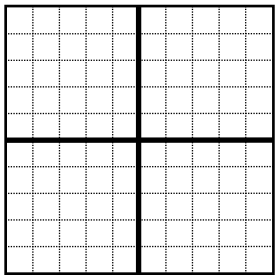
2) Complete the table for the function and find the indicate limit

2) $\lim_{x\to 0}\dfrac{\cos x-1}{x^2-x}$

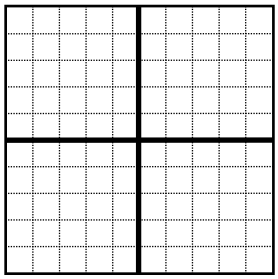
x						
$f(x)$						

Graph the following functions and use your graph to find the indicated limit

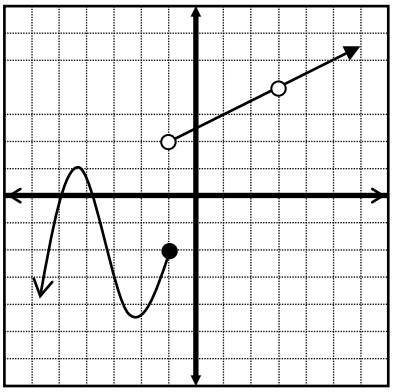
3) $f(x)=\begin{cases} 3x-1, & x<2 \\ -x^2+1, & x\geq 2 \end{cases}$ $\lim_{x\to 2} f(x)=$ _____



4. $f(x)=\begin{cases} |x-2|, & x\leq 1 \\ 1, & x>1 \end{cases}$ $\lim_{x\to 1} f(x)=$ _____



5. Use the graph to the right to find the following. (Assume that all tick marks are 1 unit apart.)



a. $\lim_{x\to 3^-} f(x) =$ _____

b. $\lim_{x\to 3^+} f(x) =$ _____

c. $\lim_{x\to 3} f(x) =$ _____

d. $\lim_{x\to -3^-} f(x) =$ _____

e. $\lim_{x\to -3^+} f(x) =$ _____

f. $\lim_{x\to -3} f(x) =$ _____

g. $\lim_{x\to 1^-} f(x) =$ _____

h. $\lim_{x\to -1^+} f(x) =$ _____

i. $\lim_{x\to -1} f(x) =$ _____

j. $f(-1) =$ _____

k. $f(3) =$ _____

6. Given $f(x) = \begin{cases} x^2 + 6 & x < 2 \\ x^3 + 2 & x \geq 2 \end{cases}$

a). $\lim_{x\to 2^-} f(x) =$

b). $\lim_{x\to 2^+} f(x) =$

c). $\lim_{x\to 2} f(x) =$

d) $f(2) =$

e). Is $f(x)$ continuous at $x = 2$? Why or why not

7. Given $f(x) = \begin{cases} x + 5 & x < 1 \\ x + 7 & x \geq 1 \end{cases}$

a). $\lim_{x\to 1^-} f(x) =$

b). $\lim_{x\to 1^+} f(x) =$

b). $\lim_{x\to 1} f(x) =$

d) $f(1) =$

e). Is $f(x)$ continuous at $x = 1$? Why or why not

Find the indicated limit.

8) $\lim_{x\to 2}\frac{x^3-2x^2+4x-8}{x^4-2x^3+x-2}$

9) $\lim_{x\to 2}\frac{x^2-4x+1}{3x-5}$

10) $\lim_{x\to 1}\frac{x^2-1}{x-1}$

11) $\lim_{x\to 0}\frac{\frac{1}{x+5}-\frac{1}{5}}{x}$

12) $\lim_{x\to 3}\frac{\frac{1}{x}-\frac{1}{3}}{x-3}$

13) $\lim_{x\to 3}\frac{x^2-x-2}{x^2-1}$

14) $\lim_{x\to 0}\frac{\sqrt{9+x}-3}{x}$

15) $\lim_{x\to 0}\frac{x}{\sqrt{x+4}-4}$

16) $\lim_{x\to 1}\frac{\sqrt{37-x}-6}{x-1}$

17) $\lim_{t\to -\infty}\frac{x^3+1}{x^3+3x^2+5}$

18) $\lim_{x\to \infty}\frac{3x-2}{5x^2+4x+1}$

19) $\lim_{x\to \infty}\frac{2x^2+1}{3x-5}$

Use the **definition of continuity** to determine whether $f(x)$ is continuous at a.

20) $f(x)=\dfrac{x-2}{x^2-4}$ $a = 2$

21) $f(x)=\dfrac{x^2-4}{x-2}$ $a = 3$

22) $f(x)=\begin{cases}5x & \text{if } x<4\\ 21 & \text{if } x=4\\ x^2+4 & \text{if } x>4\end{cases}$, $a = 4$

23) $f(x)=\begin{cases}2x & \text{if } x\le 0\\ x^2+1 & \text{if } 0<x\le 2\\ 7-x & \text{if } x>2\end{cases}$, $a = 2$

Determine for what numbers, if any, the given function is discontinuous. Use the **definition of continuity** to show the discontinuity.

24) $f(x)=\dfrac{x+1}{x^2-3x-4}$

25) $f(x)=\begin{cases}\dfrac{x^2-1}{x-1} & \text{if } x\neq 1\\ 2 & \text{if } x=1\end{cases}$

A piecewise function is given. Use the **properties of limits** to find the indicated limits, or state that a limit does not exist.

26) $f(x)=\begin{cases}1-x & \text{if } x<1\\ 2 & \text{if } x=1\\ x^2+3 & \text{if } x>1\end{cases}$ a) $\lim\limits_{x\to 1^-} f(x)$ b) $\lim\limits_{x\to 1^+} f(x)$ c) $\lim\limits_{x\to 1} f(x)$

27) $f(x)=\begin{cases}\dfrac{x^2-25}{x+5} & \text{if } x\neq 5\\ 13 & \text{if } x=5\end{cases}$ a) $\lim\limits_{x\to 5^-} f(x)$ b) $\lim\limits_{x\to 5^+} f(x)$ c) $\lim\limits_{x\to 5} f(x)$

Use the definition of the derivative to answer the following.

28) $f\text{ ‘ }(a)$ of $f(x)=3x^2-x+1$.

29) Find f ‘ (3) on $f(x)=\dfrac{3}{x}$.

Answers to Unit-1 Test Review

1) DNE
2) 0
3) DNE
4) 1
5) a. $\lim_{x\to 3^-} f(x) =$ 4
g. $\lim_{x\to 1^-} f(x) =$ 3
b. $\lim_{x\to 3^+} f(x) =$ 4
h. $\lim_{x\to -1^+} f(x) =$ 2
c. $\lim_{x\to 3} f(x) =$ 4
i. $\lim_{x\to -1} f(x) =$ DNE
d. $\lim_{x\to -3^-} f(x) =$ -3
j. $f(-1) =$ -2
e. $\lim_{x\to -3^+} f(x) =$ -3
k. $f(3) =$ undefined
f. $\lim_{x\to -3} f(x) =$ -3

6) a). $\lim_{x\to 2^-} f(x) = 10$
b). $\lim_{x\to 2^+} f(x) = 10$
c). $\lim_{x\to 2} f(x) = 10$
d) $f(2) = 10$
e). Is $f(x)$ continuous at $x = 2$? Why or why not
Yes because $\lim_{x\to 2} f(x) = f(2) = 10$

7) a). $\lim_{x\to 1^-} f(x) = 6$
b). $\lim_{x\to 1^+} f(x) = 8$
c). $\lim_{x\to 1} f(x) =$ DNE
d) $f(1) = 8$
e). Is $f(x)$ continuous at $x = 1$? Why or why not
No because $\lim_{x\to 1} f(x) =$ DNE

8) $\frac{8}{9}$
9) -3
10) 2
11) $-\frac{1}{25}$
12) $-\frac{1}{9}$
13) $\frac{1}{2}$
14) $\frac{1}{6}$
15) 0
16) $-\frac{1}{12}$
17) 1
18) 0
19) ∞

20) $f(x)$ is discontinuous at $x = 2$
21) $f(x)$ is continuous at $x = 3$
22) $f(x)$ is discontinuous at $x = 4$
23) $f(x)$ is continuous at $x = 2$
24) $f(x)$ is discontinuous at $x = -1, 4$
25) $f(x)$ is continuous at all values of x

26) a) 0 b) 4 c) DNE
27) a) 0 b) 0 c) 0
28) $f'(a) = 6a - 1$
29) $f'(3) = -\frac{1}{3}$