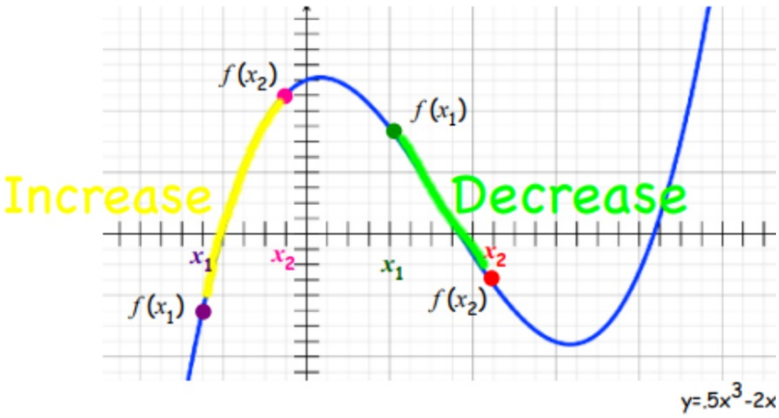
Increasing & Decreasing Functions and the First Derivative Test

Definitions of Increasing and Decreasing Functions

A function f is increasing on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is decreasing on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.



Test for Increasing and Decreasing Functions

Let f be a function that is continuous on the closed interval [a , b] and differentiable on the open interval (a , b).

1. If $f'(x) > 0$ for all x in (a , b), then f is increasing on [a , b].

2. If $f'(x) < 0$ for all x in (a , b), then f is decreasing on [a , b].

3. If $f'(x) = 0$ for all x in (a , b), then f is constant on [a , b].

Guidelines for Finding Intervals on Which a Function is Increasing or Decreasing:

1. Locate the critical numbers (zeros of f ' and any points of discontinuity) of f to determine the test intervals.
2. Determine the sign of f ' (x) at one test value in the interval.
3. Then determine if f is increasing or decreasing.

The First Derivative Test

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows:

1. If $f'(x)$ changes from negative to positive, then f has a relative minimum.

2. If $f'(x)$ changes from positive to negative, then f has a relative maximum.

3. If $f'(x)$ is positive (or negative) on both sides of c then $f(c)$ is neither a relative minimum nor a relative maximum. **(point of inflection)**

Example:

1) find the critical numbers of f (if any)
2) find the intervals on which the function is increasing or decreasing
3) apply the First Derivative Test to identify all relative extrema

$$f(x) = \frac{x^3}{4} - 3x$$

Test Intervals			
Sign of $f'(x)$			
Conclusion			