

Warm Up

According to Hindu legend, Lord Krishna once appeared as a sage before the king who ruled in a region of India and challenged him to a game of chess. The prize if Lord Krishna won was based on the chessboard: the king would provide him with a single grain of rice for the first square, two grains of rice for the second square, four grains of rice for the third square, and so on, doubling the rice with each successive square on the board until all 64 squares were filled. Lord Krishna of course did win but the king was unable to pay his debt. Why?

Exponents are a short-hand for writing multiplication of the same number

$$(6)(6) = 6^2$$

$$a \cdot a \cdot a \cdot a = a^4$$

We use the words "base" and "power"

$$2^3 \rightarrow 2 \text{ is the base, and } 3 \text{ is the power}$$

Product Rule

$x^3 \cdot x^4$ ← Same base, x , add the exponents, $3 + 4 = 7$

$$x^3 \cdot x^4 = x^7$$

$$x^3 \cdot x^4$$

$$x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^7$$

Quotient Rule

$$\frac{x^5}{x^2} \quad \leftarrow \text{Same base, } x, \text{ subtract the exponents, } 5 - 2 = 3$$

$$\frac{x^5}{x^2} = x^3$$

$$\frac{x^5}{x^2} = \frac{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x}} = x^3$$

zero exponent rule:

$$3^0 = 1$$

$$a^0 = 1$$

$$\frac{y^3}{y^3} = y^{3-3} = y^0$$
$$\frac{y^3}{y^3} = \frac{\cancel{y} \cdot \cancel{y} \cdot \cancel{y}}{\cancel{y} \cdot \cancel{y} \cdot \cancel{y}} = 1$$

The image shows two handwritten equations in purple ink. The first equation is $\frac{y^3}{y^3} = y^{3-3} = y^0$. The second equation is $\frac{y^3}{y^3} = \frac{\cancel{y} \cdot \cancel{y} \cdot \cancel{y}}{\cancel{y} \cdot \cancel{y} \cdot \cancel{y}} = 1$. Red arrows point from the y^0 in the first equation to the 1 in the second equation, indicating that $y^0 = 1$.

Power Rule

$$(y^6)^2 \leftarrow \text{Multiply the exponents, } 6 \times 2 = 12$$

$$(y^6)^2 = y^{12}$$

$$(y^6)^2 = y^6 \cdot y^6 = y^{6+6} = y^{12}$$

Negative Exponents

x^{-2} ← Get the reciprocal, or move the negative exponent down.

$$x^{-2} = \frac{x^{-2}}{1} = \frac{1}{x^2}$$

$5y^{-3}$ ← Get the reciprocal of only the base with the negative exponent, the number stays in its place.

$$5y^{-3} = \frac{5y^{-3}}{1} = \frac{5}{y^3}$$

$\frac{a^2}{b^{-4}}$ ← Get the reciprocal of the base with the negative exponent, the base with the positive exponent stays in its place.

$$\frac{a^2}{b^{-4}} = \frac{a^2 \cdot b^4}{1} = a^2 \cdot b^4$$

$$5x^{-3}y^2 = \frac{5}{1} \cdot \frac{x^{-3}}{1} \cdot \frac{y^2}{1} = \frac{5}{1} \cdot \frac{1}{x^3} \cdot \frac{y^2}{1}$$

$$= \frac{5y^2}{x^3}$$

$$\frac{y^{-4}z^3}{6x^{-2}} = \frac{x^2z^3}{6y^4} \quad \bigg| \quad \frac{1}{5^{-1}} = 5$$

Rational Indices

A rational indice (or indices) can be written as a fraction:

$$a^{1/2} = \sqrt{a}$$

-OR-

$$16^{1/4} = \sqrt[4]{16} = 2 \text{ or } -2$$

$$x^{1/4} + x^{1/4} + x^{1/4} = \sqrt[4]{x} + \sqrt[4]{x} + \sqrt[4]{x}$$

Things to remember:

$$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$$

$$\left(\frac{x^3}{y^2}\right)^4 = \frac{(x^3)^4}{(y^2)^4} = \frac{x^{12}}{y^8}$$

! Be careful: $\left(\frac{a+b}{c+d}\right)^2$ is not the same as $\frac{a^2+b^2}{c^2+d^2}$ **!**

In fact:
$$\left(\frac{a+b}{c+d}\right)^2 = \left(\frac{a^2+2ab+b^2}{c^2+2cd+d^2}\right)$$

Things to remember:

$$-(3)^4 = -(3)(3)(3)(3) = -(81) = -81$$

but...

$$(-3)^4 = (-3)(-3)(-3)(-3) = 81$$

Your calculator will always do the first way unless you tell it otherwise (by using parenthesis)

$$(3x^2)(3x^2y)^3$$

$$(3x^2)3^3(x^2)^3y^3$$

$$3x^2 \quad 27x^6y^3$$

$$81x^{2+6}$$

$$y^3 = 81x^8y^3$$