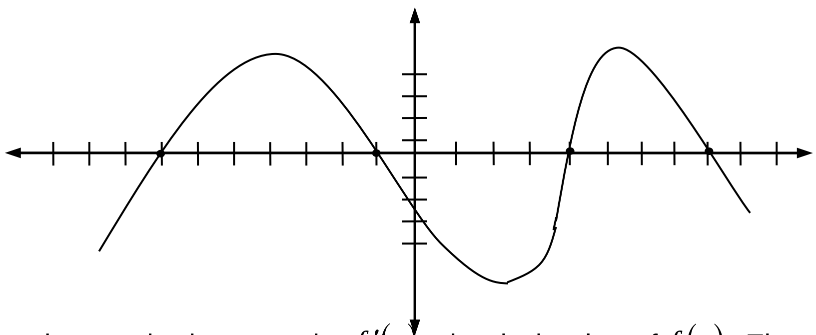
Curve Sketching Review 2

1) Consider the curve defined by the equation $y^3 + 3x^2 y + 13 = 0$.

   a. Find $\frac{dy}{dx}$.

   b. Write an equation for the line tangent to the curve at $(2,-1)$.

   c. Find the minimum $y-coordinate$ of any point on the curve. Justify your answer.



2) Consider the graph above as the $f'(x)$, the derivative of $f(x)$. The domain of the function $f(x)$ is the set of all x such that $-10 \le x \le 10$. The graph of $f'(x)$ has a zero slope when $x = -4, 2, 5$.

   a. For what values of x does the graph of f have a horizontal tangent?

   b. For what values of x in the interval $(-10,10)$ does f have a relative maximum. Justify.

   c. For what values of x is the graph concave downward? Justify your answer.

3) Consider the curve given by $y^2 = 2 + xy$.

   a. Show that $\frac{dy}{dx} = \frac{y}{2y - x}$

   b. Find all the points (x, y) on the curve where the line tangent has slope $\frac{1}{2}$

   c. Show that there are no points (x, y) on the curve where the line tangent is horizontal.

   d. Let x and y be functions of time that are related by the equation $y^2 = 2 + xy$.

   At time t = 5, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time t = 5.

4) Let f be a function that is even and continuous on the closed interval [-3, 3].
The function f and its derivatives have the properties indicated in the table below.

x	0	0 < x < 1	1	1 < x < 2	2	2 < x < 3
$f(x)$	1	POS	0	NEG	- 1	NEG
$f'(x)$	undefined	NEG	0	NEG	undefined	POS
$f''(x)$	undefined	POS	0	NEG	undefined	NEG

   a. Find the x-coordinate of each point at which f attains an absolute maximum value or an absolute minimum value. Justify your answers.

   b. Find the x-coordinate of each point of inflection on the graph of f. Justify your answers.

   c. Sketch the graph of a function with the given characteristics of f.

5) Let f be the function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

a. On what intervals is f increasing?
b. On what intervals is f concave downward?
c. Find the value of k for which f has 11 as its relative minimum.

6) Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.

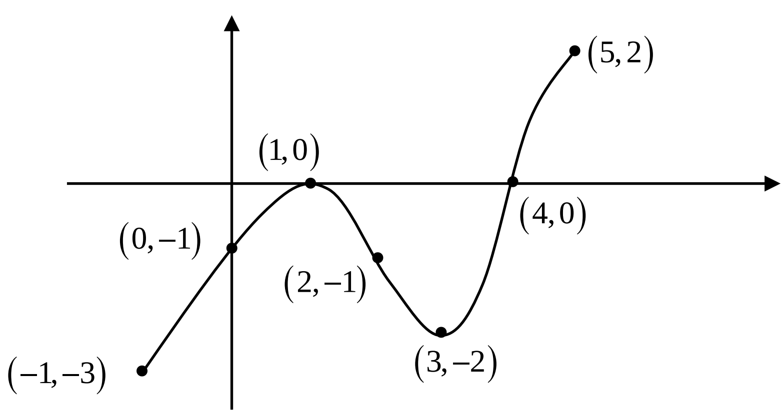
a. Find $\frac{dy}{dx}$ in terms of y.
b. Write an equation for each vertical tangent to the curve.
c. Find $\frac{d^2y}{dx^2}$ in terms of y.

7) Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$

a. Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$
b. Show that there is a point on the curve P, where the x-coordinate is 3 at which the line tangent to the curve at point P is horizontal. Find the y-coordinate.
c. Find the value of $\frac{d^2y}{dx^2}$ at point P (from part b). Does the curve have a local maximum, local minimum or neither at point P? Why?

8) The figure below shows the graph of $f'(x)$, the derivative of $f(x)$. Using the domain for $f(x)$ is $-1 \le x \le 5$, where $f'(x)$ graph has a horizontal tangent at $x = 1$, and $x = 3$. Given $f(2) = 6$.

a. Find the x-coordinate of each point of inflection of $f(x)$. Give a reason.
b. Where does $f(x)$ have and absolute minimum? Where does $f(x)$ have an absolute maximum?
c. If $g(x) = x \cdot f(x)$, find the equation of the line tangent to $g(x)$ at x = 2.



Curve Sketching Review #2

1A) $\frac{dy}{dx} = \frac{-6xy}{3y^2 + 3x^2} = \frac{-2xy}{y^2 + x^2}$

1B) $y + 1 = \frac{4}{5}(x - 2)$

1C) if $\frac{dy}{dx} = \frac{-6xy}{3y^2 + 3x^2} = \frac{-2xy}{y^2 + x^2}$ =0 then either x = 0 or y = 0… $y \neq 0$,

therefore x=0 in the original equation when $y = \sqrt[3]{-13}$

2A) $f(x)$ has a horizontal tangent when x = -7, -1, 4, 8

2B) $f(x)$ has a relative maximum when x = -1 and 8, because $f'(x)$ values change from positive to negative

2C) $f(x)$ is concave down when the slopes of $f'(x) < 0$ (-4, 2) U (5,10)

3A)

$$2y\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$(2y - x)\frac{dy}{dx} = y$$

$$\frac{dy}{dx} = \frac{y}{(2y - x)}$$

3B) when $\frac{dy}{dx} = \frac{y}{(2y - x)}$ = ½ when x= 0 and from the original equation $y = \pm\sqrt{2}$,

so…answers are (0, $\sqrt{2}$) and (0, -$\sqrt{2}$)

3C) if the curve is horizontal, $\frac{dy}{dx} = \frac{y}{(2y - x)}$ =0 therefore y = 0. $y \neq 0$ from the original equation,

therefore there are no horizontal tangents

3D)

$$2y\frac{dy}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$2(3)(6) = \left(\frac{7}{3}\right)(6) + (3)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \left(\frac{22}{3}\right)$$

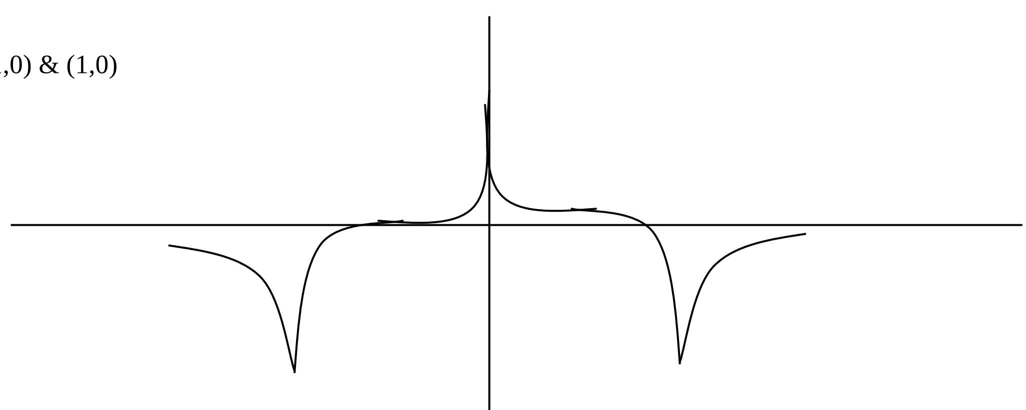
4A) EVEN FUNCTIONS SYMMETRIC OVER y-axis

ABSOLUTE MAX @ x = 0

ABSOLUTE MIN @ x = -2, 2

4B) Point of inflection @ (-1,0) & (1,0)

4C)



5A) $\left(-\infty,\frac{1}{3}\right)\cup(3,\infty)$ $f'(x)$ values are positive

5B) $\left(-\infty,\frac{5}{3}\right)$ $f''(x)$ values are negative

5C) from Part A, relative min exists if and only if x = 3, therefore $f(3)=11=3^3-5\bullet3^2+3\bullet3+k;k=20$

6A) $\frac{dy}{dx}=\frac{1}{1-\sin y}$

6B) vertical line has undefined slope, therefore, if 1-sin y = 0, then $\frac{dy}{dx}$ is undefined.

When y = $\frac{\pi}{2}$ the equation of the vertical line is x = $\frac{\pi}{2}$ - 1

6C) $\frac{d^2y}{dx^2}=\frac{\cos y}{(1-\sin y)^2}\frac{dy}{dx}=\frac{\cos y}{(1-\sin y)^3}$

7A) True

7B) find both in either order: x = 3; $x^2+4y^2=7+3xy$; dy/dx = 0.
Point P (3,2) → y –coordinate = 2.

C) because dy/dx = 0, there is a horizontal tangent line. Must use concavity (second derivative) to find if it's a max, min or neither at (3,2). (3,2) is a max.

8A) x = 1, 3 because the slopes of f'(x) (i.e. f''(x)) changes signs.

8B) Abs. min at x = 4 ; Abs. max at x= -1

8C) y-12 = 4 (x – 2)