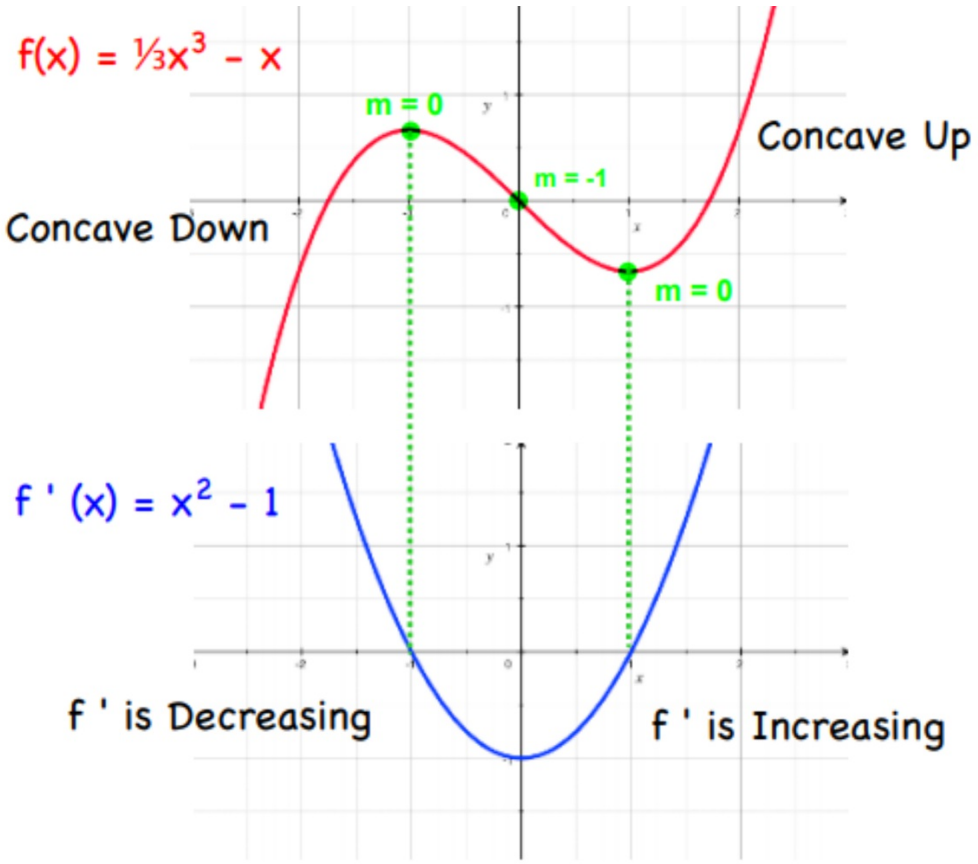
Concavity and the Second Derivative Test

Definition of Concavity:

Let f be differentiable on an open interval I.

The graph of f is **concave upward** on I if f' is **increasing** on the interval.

The graph of f is **concave downward** on I if f' is **decreasing** on the interval.



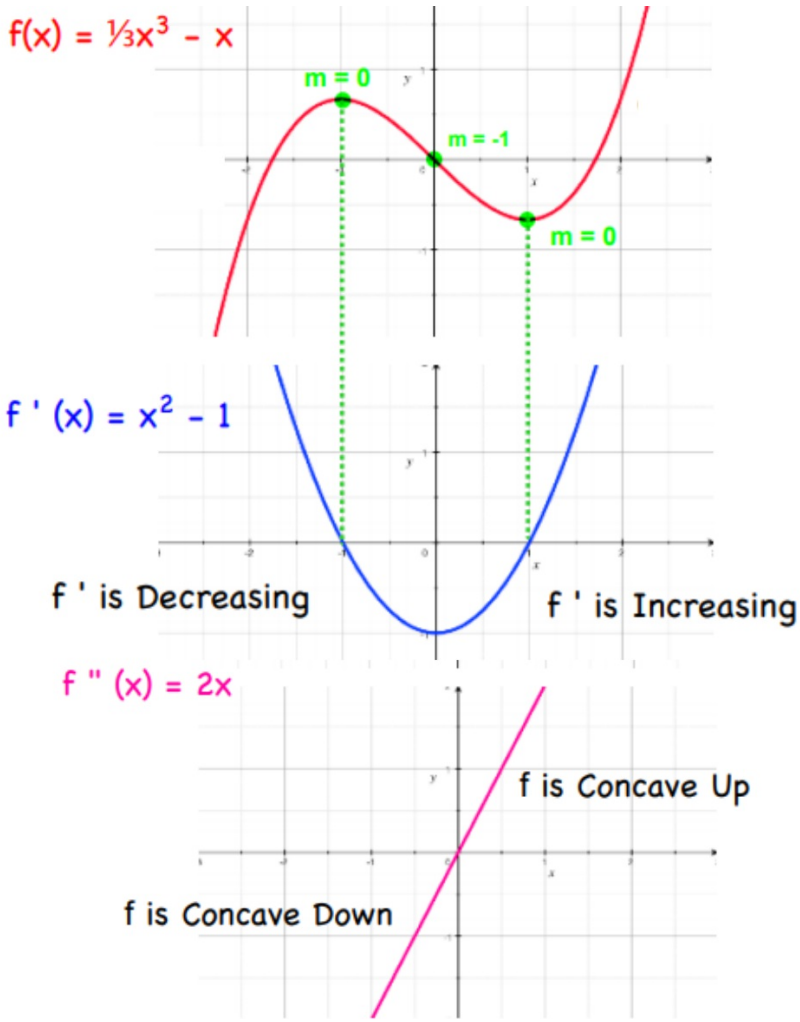
The concavity of f is related to the slope of the derivative.

Test for Concavity:

Let f be a function whose second derivative exists on an open interval I.

1.) If $f''(x) > 0$ for all x in I, then the graph of f is **concave upward** in I.

2.) If $f''(x) < 0$ for all x in I, then the graph of f is **concave downward** in I.


f(x) = ⅓x³ − x
m = 0
m = -1
m = 0
f ' (x) = x² − 1
f ' is Decreasing
f ' is Increasing
f " (x) = 2x
f is Concave Up
f is Concave Down

Points of Inflection

If (c , f (c)) is a point of inflection on the graph of f, then either f "(c) = 0 or f "(c) does not exist at x = c.

Informal Definition of the Points of Inflection:

the point of the graph when the concavity changes from up to down (or down to up)

Second Derivative Test

Let f be a function such that $f''(c) = 0$ and the second derivative of f exists on an open interval containing c.

1) If $f''(c) > 0$, the f has a relative minimum at $(c , f(c))$.

2) If $f''(c) < 0$, the f has a relative maximum at $(c , f(c))$.

If $f''(c) = 0$, the test fails. That is, f may have a relative maximum, a relative minimum, or neither. In such cases, you can use the First Derivative Test.

Example: Determine the open intervals on which the graph of the given function is concave up or concave down.

$$f(x) = \tfrac{1}{4}x^4 - 2x^2$$

1. Find the first derivative of f and find the critical numbers

2. Find the second derivative and apply the second derivative test

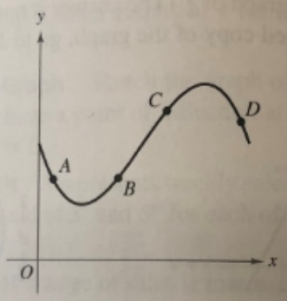
Point			
Sign of f"(x)			
Conclusion			

Example: Find the relative extrema of the following function.

$$f(x) = -3x^5 + 5x^3$$

Calculus AP® – Exam Preparation Questions

87. Multiple Choice At which of the four points on the graph shown is dy/dx positive and d^2y/dx^2 negative?



(A) A (B) B

(C) C (D) D

88. Multiple Choice The graph of the function $h(x) = 4xe^{-x}$ is

(A) decreasing and concave upward on $(2, \infty)$.

(B) increasing and concave downward on $(-\infty, 2)$.

(C) increasing and concave upward on $(2, \infty)$.

(D) decreasing and concave upward on $(1, \infty)$.

89. Free Response Consider the function $f(x) = 2 \sin x - x$ on $[0, 2\pi]$.

(a) Find $f'(x)$ and $f''(x)$.

(b) What are the critical numbers of f?

(c) Find all relative extrema of f.

(d) How many point(s) of inflection does the graph of f have? Where do they occur?