L.T.: I can use the binomial theorem to expand polynomials

Expand:

$$(a+b)^0 = 1$$

$$(a+b)^1 = a+b$$

$$(a+b)^2 = (a+b)(a+b)$$

$$= a^2 + 2ab + b^2$$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 +$

L.T.: I can use the binomial theorem to expand polynomials

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 +$

$(a+b)^6 =$

$a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 +$

$6ab^5 + b^6$

L.T.: I can use the binomial theorem to expand polynomials

See how I'm doing it?

Four things are going on:

1. **The a guys start at the original power and step down one each time...**

$$a^6,\ a^5,\ a^4,\ a^3,\ a^2,\ a^1,\ a^0 \leftarrow \text{no } a$$

2. **The b guys do the opposite...**

$$\text{no } b \rightarrow b^0,\ b^1,\ b^2,\ b^3,\ b^4,\ b^5,\ b^6$$

3. **The powers on all the ab sets add up to the original power...**

$$\overset{6}{a^6} \quad \overset{6}{a^5b^1} \quad \overset{6}{a^4b^2} \quad \overset{6}{a^3b^3} \quad \overset{6}{a^2b^4} \quad \overset{6}{a^1b^5} \quad \overset{6}{b^6}$$

4. **What about the coefficients?**

1, 6, 15, 20, 15, 6, 1

What's going on there?

L.T.: I can use the binomial theorem to expand polynomials

Let's write them all out to see if we can find a pattern...

$$
\begin{array}{c}
1 \leftarrow (a+b)^0 \\
1 \quad 1 \leftarrow (a+b)^1 \\
1 \quad 2 \quad 1 \leftarrow (a+b)^2 \\
1 \quad 3 \quad 3 \quad 1 \leftarrow (a+b)^3 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \leftarrow (a+b)^4 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \leftarrow (a+b)^5 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \leftarrow (a+b)^6 \\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1
\end{array}
$$

L.T.: I can use the binomial theorem to expand polynomials

What's the trick?

What's the next line?

This is called Pascal's Triangle.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

Ones on the edge... add the two guys above!

Whoa!

L.T.: I can use the binomial theorem to expand polynomials

October 27, 2017

Warm Up

Write out the expansion of $(a + b)^7$

L.T.: I can use the binomial theorem to expand polynomials

Let's expand $(x+2)^4$

From Pascal's triangle, we'll use these coefficients:

$$1,\ 4,\ 6,\ 4,\ 1$$

Here we go!

$$(x+2)^4 = 1x^4 + 4x^3(2)^1 + 6x^2(2)^2 + 4x^1(2)^3 + 1(2)^4$$

Since this is a number, we'll have some clean-up.

$$= x^4 + 8x^3 + 24x^2 + 32x + 16$$

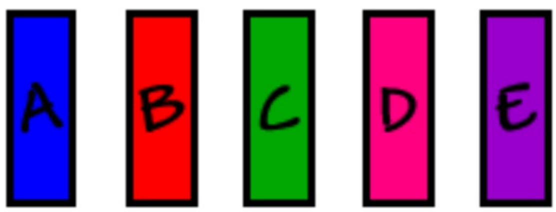
L.T.: I can use the binomial theorem to expand polynomials

They just get messier -- not much worse.

Expand $(2x-3)^5$

Look at it like $((2x)+(-3))^5$

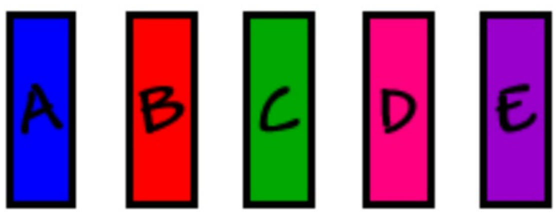
Now, suppose you have 5 books to arrange on a bookshelf. How many different ways could you do it?



We call this a factorial and we write: 5!

Now you have your 5 books but only 3 spots on the shelf, how many different combinations could you have?

note: the order you put up the books does NOT matter -- this means book combination ABC is the same as BAC



___ ___ ___

In general, if you have n books and r spots on the bookshelf, how many different ways can you arrange them?

The number of ways to CHOOSE n objects taken r at a time is

$$_{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

*** This can also be phrased as "how many combinations?"**

Notation:

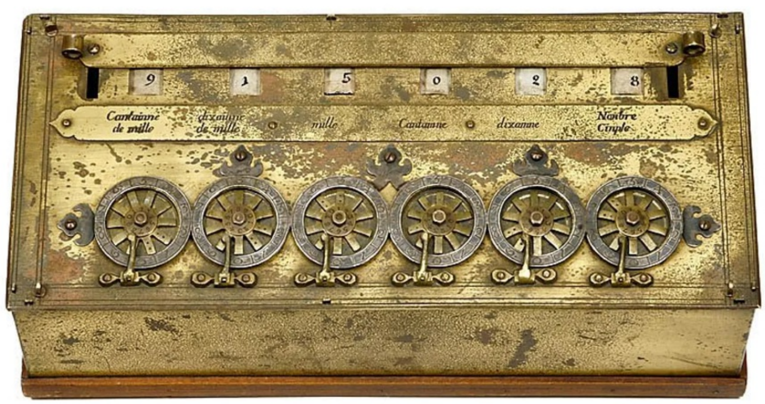
$$ {}_{n}C_{r} = C^{n}_{r} = C(n, r) = \binom{n}{r} $$

"C" is for "choose"
(it's really "combinations")

This is NOT
a fraction!

So why am I bothering to talk about this stuff when we're supposed to be doing binomial expansion and talking about Pascal's triangle???

Blaise Pascal lived in France in the 1600s. He was a mathematician, philosopher, weatherman (he did some of the first work with barametric pressure), physist, and inventor. He came up with the earliest automated calculator, called a pascaline, in fact, Turing sited Pascal as the true father of modern computing!



Where we become a little more interested in Pascal is the fact, despite being a devout catholic, he had a bit of a gambling problem. He corresponded with Fermat (awesome mathematician that we'll talk more about soon) about the odds of winning while gambling and wrote the begining work on probability and combinations. He developed Pascal's triangle from these combinations.

Take a look at this, what do each of these equal??

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

$$\binom{5}{0} \quad \binom{5}{1} \quad \binom{5}{2} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}$$

So,

$$(x+y)^3 = \binom{3}{0}x^3y^0 + \binom{3}{1}x^2y^1$$

$$+ \binom{3}{2}x^1y^2 + \binom{3}{3}x^0y^3$$

$$= \frac{3!}{3!\,0!}x^3y^0 + \frac{3!}{2!\,1!}x^2y^1 +$$

$$+ \frac{3!}{1!\,2!}x^1y^2 + \frac{3!}{0!\,3!}x^0y^3$$

Look at these and look at the exponents!

$$= x^3 + 3x^2y + 3xy^2 + y^3$$

For the expansion of $(a + b)^{n}$

$$\frac{n!}{(n-r)!\,r!}\,a^{n-r}\,b^{r}$$

Let's find the 13th term in the expansion of $(a+b)^{15}$... Without pathetically trying to multiply it all out!

OK, think about it...

The second variable... the b...

The b starts at b^0 and goes up one each time... So, at the 13th term, it will be

$$b^{12}$$

No more thinking left to do now!

These have to add up to $(a+b)^{15}$

$$a^{\_}\ b^{12}$$

$$a^3 b^{12}$$

Now, pop the coefficient:

$$\frac{15!}{3!\ 12!} a^3 b^{12} = 455 a^3 b^{12}$$

Homework:
Page 195, Section 7B, Problems 1, 3, 5, 7, 8
Due Monday (10/30/17)

1 Write down the first three and last two terms of the binomial expansion of:

a $(1+2x)^{11}$ **b** $\left(3x+\frac{2}{x}\right)^{15}$ **c** $\left(2x-\frac{3}{x}\right)^{20}$

3 Find the coefficient of:

a x^{10} in the expansion of $(3+2x^2)^{10}$ **b** x^3 in the expansion of $\left(2x^2-\frac{3}{x}\right)^6$

c x^6y^3 in the expansion of $\left(2x^2-3y\right)^6$ **d** x^{12} in the expansion of $\left(2x^2-\frac{1}{x}\right)^{12}$.

5 **a** Write down the first 6 rows of Pascal's triangle.

b Find the sum of the numbers in:

i row 1 **ii** row 2 **iii** row 3 **iv** row 4 **v** row 5.

c Copy and complete:

The sum of the numbers in row n of Pascal's triangle is ......

d Show that $(1+x)^n = \binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+ \dots + \binom{n}{n-1}x^{n-1}+\binom{n}{n}x^n$.

Hence deduce that $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+ \dots + \binom{n}{n-1}+\binom{n}{n}=2^n$.