Given some polynomial

$$f(x) = \frac{ax^n + \ldots}{bx^m + \ldots}$$

← nth degree polynomial

← mth degree polynomial

1. If $n < m$, then the x-axis is the horizontal asymptote.

2. If $n = m$, then the horizontal asymptote is the line $y = \frac{a}{b}$

3. If $n > m$, then there is no horizontal asymptote. (There is a slant diagonal or oblique asymptote.)

$$f(x) = \frac{1x^2 - x - 6}{1x^2 - 1}$$

← degree of 2

← degree of 2

$\frac{a}{b} = \frac{1}{1} = 1$ ← horiz. $y = 1$

Vert: $x^2 - 1 = 0$

$+1 \quad +1$

$x^2 = 1$

$\sqrt{} \quad \sqrt{}$

$x = 1$ and $x = -1$

State all asymptotes:

$$f(x) = \frac{x-3}{x^2-3x-10}$$

← degree 1

← degree 2

horiz at x-axis $y=0$

vert: $x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=5 \quad x=-2$

Graph:

$$y = \frac{2x + 5}{x - 1}$$

Proceedure

1. Find any vertical asymptotes
2. Find any horizontal asymptotes
3. Find any x-intercepts
4. Find any Y-intercepts
5. Find a few points on both sides of the vertical asymptote
6. Be happy you can now graph rational functions! ☺

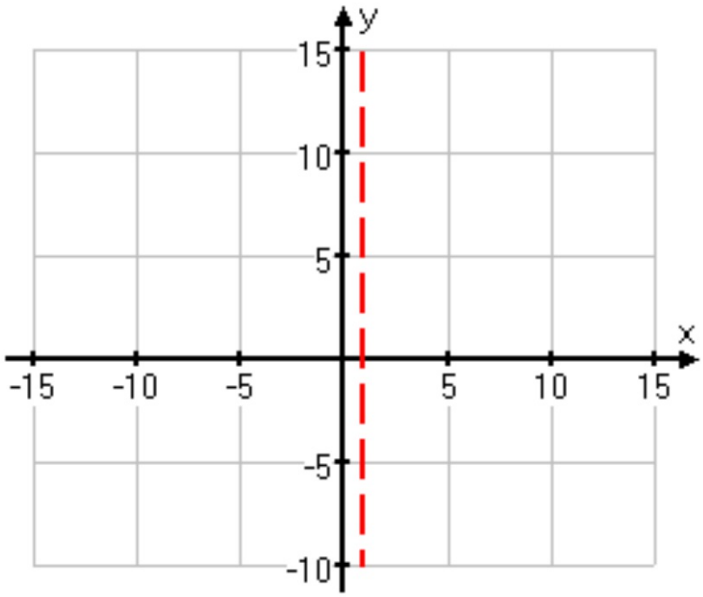
Vertical Asymptote(s):

$$y = \frac{2x + 5}{x - 1}$$

$x - 1 = 0$

$+1 \quad +1$

$x = 1$



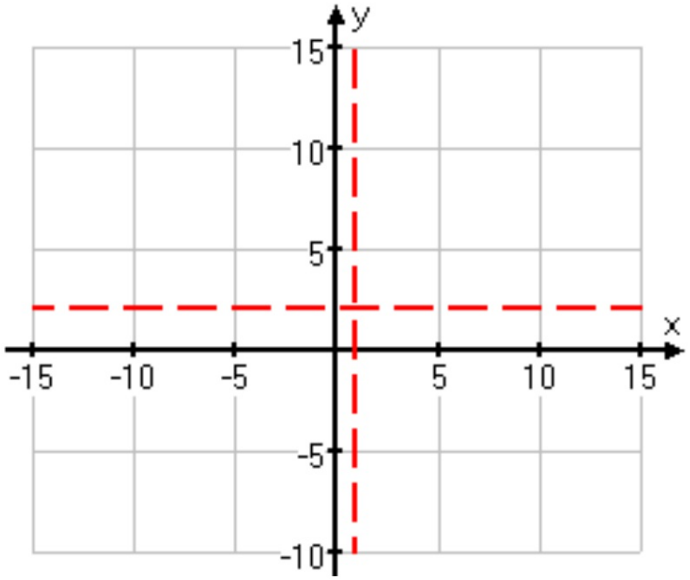
Horizontal Asymptote(s):

$$y = \frac{2x + 5}{1x - 1}$$

← deg. 1
← deg. 1

$y = \frac{a}{b}$

$= \frac{2}{1} = 2$



$$y = \frac{2x + 5}{x - 1}$$

x-intercept(s):

$0 = \frac{2x+5}{x-1}$ $\frac{2x=-5}{2\quad 2}$

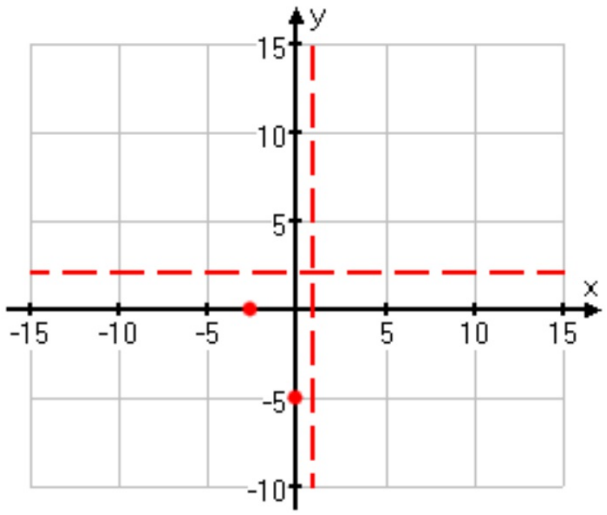
$\cdot(x-1)\quad \cdot(x-1)$ $x = -2.5$

$0 = 2x+5$

$-5 \quad -5$

y-intercept(s):

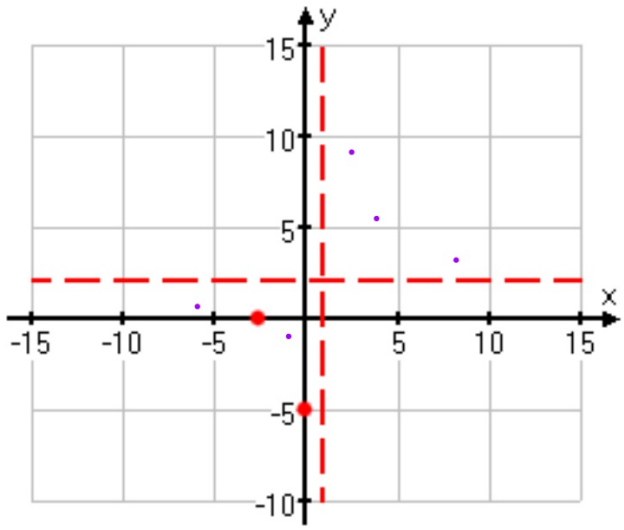
$\frac{2\cdot 0+5}{0-1} = \frac{5}{-1} = -5$

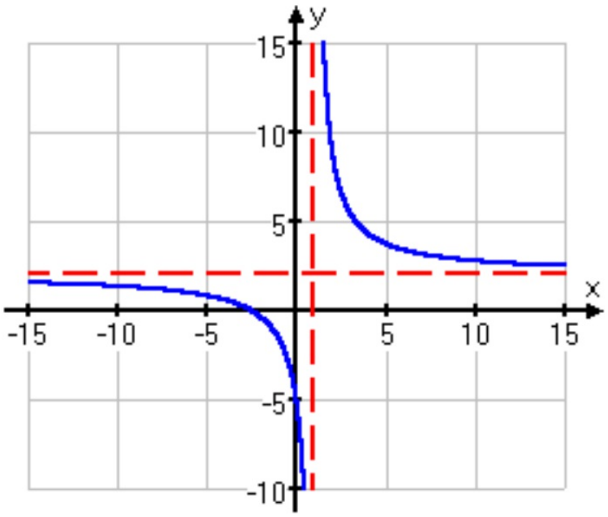


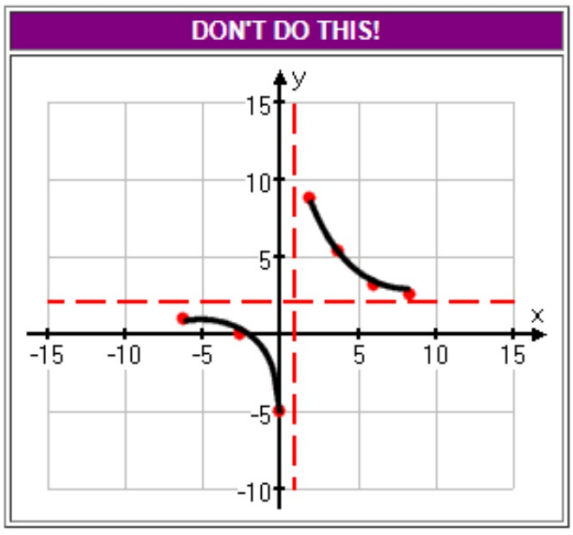
$$y = \frac{2x + 5}{x - 1}$$

Find a few more points:

x	y
-6	1
2	9
-1	-1.5
3	5.5
4	3




y
15
10
5
x
-15
-10
-5
5
10
15
-5
-10


DON'T DO THIS!
y
15
10
5
x
-15
-10
-5
5
10
15
-5
-10

You try:

Due tomorrow (12/6/17)
Page 62 Section 1G Problem 1

Plus go back and see if you can now problems 5 and 8 from the packet

EXERCISE 1G

1 For the following functions:

- **i** determine the equations of the asymptotes
- **ii** discuss the behaviour of the function as it approaches its asymptotes
- **iii** find the axes intercepts
- **iv** sketch the graph of the function.

a $f : x \mapsto \dfrac{3}{x-2}$ **b** $y = 2 - \dfrac{3}{x+1}$ **c** $f : x \mapsto \dfrac{x+3}{x-2}$ **d** $f(x) = \dfrac{3x-1}{x+2}$