Related Rates and Optimization Review #2

1) A 17 foot ladder is leaning against a wall. It is sliding down the wall at a rate of 2 ft/min.
   A) How fast is the ladder moving a away from the wall when the top is 15 ft from the ground?
   B) How fast is the area enclosed by the ladder changing at that time?
   C) How fast is the angle with the ground changing at that time?

2) A cone with a diameter of 14 ft and a height of 28 ft is being filled with oil at a rate of 2 ft^3 per second.
   A) Express the volume of the water as a function of the water level (h).
   B) How fast is the level of the oil rising in the cone when the height reaches 10 ft?
   C) How fast is the radius increasing at that time?
   D) How fast is the exposed surface area increasing at that time?

3) A man observes the launching of a rocket from a distance of 300 ft. The rocket is launched at the speed of 100 ft/s.
   A) How fast is the rocket moving away from the man 4 seconds after launch?
   B) How fast is the angle changing at this same moment?

4) A spherical balloon is being inflated at the rate of 2 in^3 per second. How fast is the radius increasing when the radius is 10 in?

5) A six foot tall man is walking towards a light pole at a speed of 2 ft per second. The light 10 ft up casts a shadow behind the man. How fast is the tip of the shadow moving toward the light pole when he is 8 ft from the light pole?

6) Melted chocolate is flowing into a giant cone at a rate of 3 ft^3 per second into a giant ice cream cone with diameter 30 ft in height 180 ft.
   A) Find the volume of the melted chocolate as a function of the chocolate level h.
   B) How fast is the level of the cone rising when the height is 12 feet?
   C) How fast is the radius changing when the height is 12 feet?

7) Determine the maximum area of a rectangle that can be enclosed with 200 meters of fencing if one side is not fenced in.

8) Determine the minimum area of a poster that will contain 50 square inches of printed material and have 4 inch margins on the top and bottom and 2 inch margins on the left and right.

9) Determine the dimensions of a box of maximum volume that can be made from a piece of material 8″ x 10″. The box is to be made by cutting square pieces from the corners and folding up the sides. The box will not have a top.

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10) Given a length of string L = 80 inches, construct a circle and a square such that the sum of the areas is a maximum. Find the maximum area of both the circle and square to the nearest tenths.

Answers:

1. a) 3.75 ft/min
   b) 20.125 ft^2/min
   c) -0.25 rad/min
2. a) $V = \frac{\pi}{48}h^3$
   b) $\frac{8}{25\pi}$ ft/sec
   c) $\frac{2}{25\pi}$ ft/sec
   d) $\frac{2}{5}$ ft^3/sec
3. a) 80 ft/sec
   b) 0.12 rad/sec
4. $\frac{1}{200\pi}$ in/sec
5. -5 ft/sec
6. a) $V = \frac{\pi}{432}h^3$
   b) $\frac{3}{\pi}$ ft/sec
   c) $\frac{1}{4\pi}$ ft/sec
7. 5000 m^2
8. 162 in^2
9. 1.47" x 5.06" x 7.06"
10. circle area is 98.5 $in.^2$
    Square area is 125.5 $in.^2$