AB Calculus Integrals as Net Change and Volume Review **Name:** _____

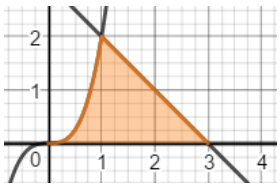
1. Find the area of the region in the first quadrant enclosed by $y = 4 - x^2$, $y = 3x$, and the y-axis.

2. Find the area of the region between the graphs of $y = \sqrt[3]{x}$ and $y = x^2 - 2x - 8$.

3. Find the volume of the solid generated by rotating the region bounded by the graphs of $y = \frac{1}{2}x$, $x = 6$, and the x-axis about the x-axis.

4. Find the volume of the solid generated by rotating the region bounded by the graphs of $x = 5 - y^2$ and $x = y - 1$ about the line $x = -4$.

5. Let R be the region bounded by the graphs of $y = 2x^3$, $y = 0$, and $y = 3 - x$.



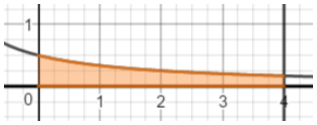
a) Find the area of region R.

b) Find the volume of the solid generated by rotating R about the x-axis.

c) Find the volume of the solid generated by rotating R about the y-axis.

d) The region R is the base of a solid. For this solid, cross sections perpendicular to the y-axis are semicircles. Find the volume of this solid.

6. Let R be the region in the first quadrant bounded by the graphs of $y = \frac{1}{x+2}$ and $x = 4$.



a) Find the volume of the solid generated by rotating R about the line $y = -1$

b) Find the volume of the solid generated by rotating R about the line $y = 3$

c) The vertical line $x = k$ divides the region into two regions of equal area. Find the value of k.

d) The region R is the base of a solid. For this solid, cross sections perpendicular to the x-axis are equilateral triangles. Find the volume of this solid.

7. Find the length of the curve $y = x\cos x$ over the interval $0 \le x \le \pi$.

8. Find the length of the curve $x = \frac{1}{2}y^2 + y - \frac{1}{2}$ from $y = -1$ to $y = 3$.

9. A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln(t^2 - 3t + 3)$. The particle is at position $x = 8$ at time $t = 0$.

a) Find the acceleration of the particle at time $t = 4$.

b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

c) Find the position of the particle at time $t = 2$.

d) Find the average speed of the particle over the interval $0 \le t \le 2$.

10. A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \le t \le 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?

b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?

c) At what time t, $0 \le t \le 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.

d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.