

AP Calculus AB Final Exam Free Response Review Problems

1) For the function $g(x) = \begin{cases} \frac{x^2+x-2}{x-1}, & x \neq 1 \\ 5, & x = 1 \end{cases}$

- a) Use limits to show that $g(x)$ is discontinuous at $x = 1$ and state why it is discontinuous there.
- b) Determine if the discontinuity is removable or non-removable and state why.

2) Find the point on $y = \sqrt{3x-1}$ where the tangent line is perpendicular to the line $3y + 2x = 3$. Then write an equation of the tangent line to the curve $y = \sqrt{3x-1}$ at that point

3) Liquid is being poured into a large vat. After t hours, the amount of gallons of liquid in the vat can be represented by $V(t) = 5t - \sqrt{t}$.

- a) What is the average rate of liquid poured into the vat over the first 4 hours ($t = 0$ to $t = 4$)? (Include units of measure)
- b) At what rate is the liquid being poured into the vat when $t = 4$? (Include units of measure)

4) A ball is thrown straight up in the air from a point 64 feet above ground level so that its position function is $h(t) = -16t^2 + 48t + 64$, where t is measured in seconds. Use this to answer the following questions. Include units with all answers.

a) What is the height of the ball at $t = 2$?

b) What is the velocity at $t = 2$?

c) When does the ball reach its greatest height?

d) What is the greatest height?

e) At what time is the ball **falling** at a speed of 48 feet per second?

f) At what time does the ball hit the ground?

g) With what velocity does the ball hit the ground?

h) What is the ball's average velocity from $t = 0$ to $t = 2$?

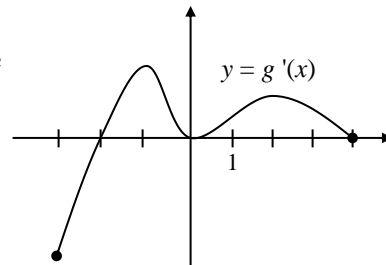
i) What is the ball's acceleration at $t = 2$?

j) Is the speed increasing or decreasing at $t = 2$?

5) Let f be the function defined by $f(x) = xe^{(1-x)}$ for all real numbers x .

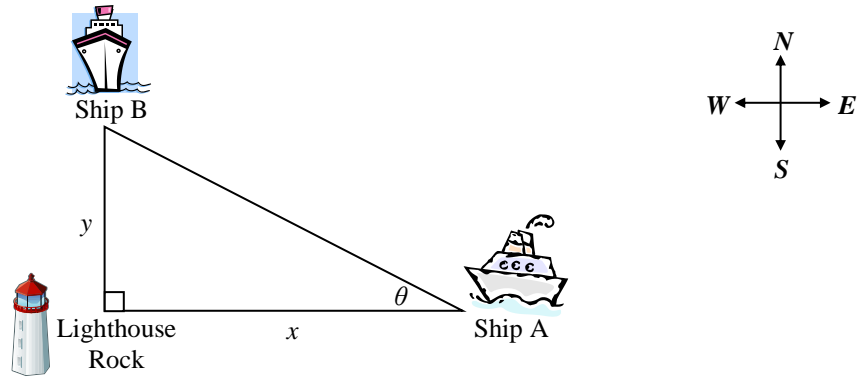
- Find each interval on which f is increasing. Justify your answer.
- Find the range of f .
- Find the each point of inflection of the graph of f . Justify your answer.
- Using the results found in parts a, b, and c, sketch the graph of f .

6) To the right is the graph of $g'(x)$, the **derivative** of a continuous function, g . The domain of g is $[-3, 4]$, **the range of g is $[-3, 2]$** , and $g(-3) = -2$, $g(0) = 0$, and $g(2) = 1$.



Find the following. Justify your answers.

- interval(s) where g is increasing
 - x-coordinate** of each rel. min
 - interval(s) where g is concave down
 - x-coordinate** of each pt. of inflection
- e) Sketch the graph of the function $y = g(x)$



- 7) Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour. Ship B is traveling due north away from Lighthouse Rock at a speed of 10 kilometers per hour. Let x be the distance between Ship A and Lighthouse Rock at time t , and let y be the distance between Ship B and Lighthouse Rock at time t , as shown in the figure above.
- Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.
 - Find the rate of change, in kilometers per hour, of the distance between the two ships when $x = 4$ km and $y = 3$ km.
 - Let θ be the angle shown in the figure. Find the rate of change of θ , in radians per hour, when $x = 4$ km and $y = 3$ km.