

AA - Matrix Review

Name: _____ Hr: _____

1. Represent the following system of equations with matrices

$$3x - 4y = 10$$

$$5y - 2x = 20$$

$$\begin{bmatrix} 3 & -4 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

OR $\begin{bmatrix} 3 & -4 & | & 10 \\ 5 & -2 & | & 20 \end{bmatrix}$

2. Multiply the following matrices: ROW · COLUMN

$$\begin{bmatrix} 2 & 3 & 7 \\ -2 & 5 & 0 \end{bmatrix} \begin{bmatrix} 7 \\ -1 \\ 2 \end{bmatrix} \begin{bmatrix} 10 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 25 & 36 \\ -19 & -5 \end{bmatrix}$$

3. Find $[A] \cdot [B]$. Show how to calculate at least two of the entries in the answer matrix by hand.

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 4 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 9 \\ -3 & 8 \\ 2 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 9 & 5 & 0 \\ 6 & -3 & 1 \end{bmatrix}$$

$$[A] \cdot [B] = \begin{bmatrix} -3 & 44 \\ -2 & 22 \end{bmatrix}$$

4. Find $2[A] - 3[C]$. Show how to calculate at least two of the entries in the answer matrix by hand.

$$2[A] - 3[C] = \begin{bmatrix} 4 & 6 & -2 \\ 0 & 8 & 10 \end{bmatrix} - \begin{bmatrix} 27 & 15 & 0 \\ 18 & -9 & 3 \end{bmatrix} = \begin{bmatrix} -23 & -9 & -2 \\ -18 & 17 & 7 \end{bmatrix}$$

5. For the matrices above, which number is A_{23} ? MATRIX [C] 1ST ROW, 2ND COLUMN = 5
 Which number is C_{12} ? MATRIX [A] 2ND ROW, 3RD COLUMN = 5

6. Solve the following matrix equation for $[x]$ using matrix inverse (by hand)

$$\begin{bmatrix} 3 & 7 \\ -6 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -34 \end{bmatrix}$$

$$[A][x] = [B]$$

$$[A]^{-1}[A][x] = [A]^{-1}[B]$$

$$[x] = [A]^{-1}[B]$$

$$[A]^{-1}[A] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 3 & 7 \\ -6 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[A]^{-1} = \begin{bmatrix} \frac{5}{57} & -\frac{7}{57} \\ \frac{6}{57} & \frac{3}{57} \end{bmatrix}$$

$$\begin{aligned} 5(3A - 6B) &= 1 & 5(3C - 6D) &= 0 \\ 6(7A + 5B) &= 0 & 6(7C + 5D) &= 1 \end{aligned}$$

$$57A = 5 \quad 57C = 6$$

$$A = \frac{5}{57} \approx 0.088$$

$$C = \frac{6}{57} \approx 0.11$$

$$\therefore B = -\frac{7}{57} \approx -0.123$$

$$\therefore D = \frac{3}{57} \approx 0.053$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{57} & -\frac{7}{57} \\ \frac{6}{57} & \frac{3}{57} \end{bmatrix} \begin{bmatrix} -2 \\ -34 \end{bmatrix}$$

$$x = \frac{-10}{57} + \frac{238}{57} = \frac{228}{57} = 4$$

$$y = \frac{-12}{57} - \frac{102}{57} = \frac{-114}{57} = -2$$

7. Use the provided matrices below and specify which combinations can be multiplied together, then indicate the dimensions (rows x columns) of the resulting matrix (don't actually calculate)

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 11 & -1 \\ 4 & 3 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 7 \end{bmatrix} \quad D = \begin{bmatrix} -1 & 0 & 4 \\ 3 & 1 & 1 \end{bmatrix} \quad v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & -1 \\ -2 & 0 \\ 1 & -2 \end{bmatrix} \quad F = \begin{bmatrix} -5 & 2 \\ -1 & 3 \\ 4 & -4 \end{bmatrix} \quad w = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 1 \\ 3 & -1 & -2 \\ 0 & 2 & -1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$x = [2 \ -1 \ 1 \ -2] \quad y = [-3 \ 2 \ 1 \ -6]$$

Handwritten notes for matrix multiplication compatibility:

- $[A] \rightarrow [B] \quad 2 \times 2$
- $[A] \rightarrow [U] \nleftrightarrow [V] \quad 2 \times 1$
- $[B] \rightarrow [U] \nleftrightarrow [V] \quad 2 \times 1$
- $[D] \nleftrightarrow [C] \rightarrow [E] \nleftrightarrow [F] \quad 2 \times 3$
- $[D] \nleftrightarrow [C] \rightarrow [W] \quad 2 \times 1$
- $[G] \nleftrightarrow [H] \rightarrow [E] \nleftrightarrow [F] \quad 3 \times 2$
- $[E] \nleftrightarrow [F] \rightarrow [U] \nleftrightarrow [V] \quad 3 \times 1$
- $[G] \nleftrightarrow [H] \rightarrow [W] \quad 3 \times 1$
- $[E] \nleftrightarrow [F] \rightarrow [U] \nleftrightarrow [V] \quad 3 \times 1$
- $[E] \nleftrightarrow [F] \rightarrow [U] \nleftrightarrow [V] \quad 3 \times 1$

8. Solve this matrix using the determinant method $[A]^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$[A]^{-1} = \frac{1}{63} \begin{bmatrix} 3 & 6 \\ -9 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -6 \\ 9 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 24 \\ 30 \end{bmatrix}$$

$$[A]^{-1} = \begin{bmatrix} \frac{3}{63} & \frac{6}{63} \\ \frac{-9}{63} & \frac{3}{63} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{63} & \frac{6}{63} \\ \frac{-9}{63} & \frac{3}{63} \end{bmatrix} \begin{bmatrix} 24 \\ 30 \end{bmatrix}$$

9. Solve for p and q:

$$\begin{bmatrix} 1 & 4 & q \\ 9 & p^2 & 2 \end{bmatrix} \begin{bmatrix} 10 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} q^2 \\ 100 \end{bmatrix}$$

$$10 + 8 + 4q = q^2$$

$$90 + 2p^2 + 8 = 100$$

$$18 + 4q = q^2$$

$$2p^2 + 98 = 100$$

$$0 = q^2 - 4q - 18$$

$$2p^2 = 2$$

$$q = \frac{4 \pm \sqrt{16 + 72}}{2}$$

$$p^2 = 1$$

$$p = \pm 1$$

$$q \approx 6.7 \quad \text{or} \quad q \approx -2.7$$

