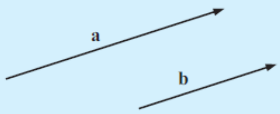
SL 1
Parallelism and Unit Vectors WS #7

Name________________________

G PARALLELISM

Two non-zero vectors are **parallel** if and only if one is a scalar multiple of the other.



- If $\mathbf{a}$ is parallel to $\mathbf{b}$, then there exists a scalar k such that $\mathbf{a} = k\mathbf{b}$.
- If $\mathbf{a} = k\mathbf{b}$ for some scalar k, then
  - $\mathbf{a}$ is parallel to $\mathbf{b}$, and
  - $|\mathbf{a}| = |k|\,|\mathbf{b}|$.

Find r and s given that $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ r \end{pmatrix}$ is parallel to $\mathbf{b} = \begin{pmatrix} s \\ 2 \\ -3 \end{pmatrix}$.

Since $\mathbf{a}$ and $\mathbf{b}$ are parallel, $\mathbf{a} = k\mathbf{b}$ for some scalar k.

$\therefore \begin{pmatrix} 2 \\ -1 \\ r \end{pmatrix} = k \begin{pmatrix} s \\ 2 \\ -3 \end{pmatrix}$ $\quad \therefore \; 2 = ks, \; -1 = 2k$ and $r = -3k$

Consequently, $k = -\frac{1}{2}$ and $\therefore \; 2 = -\frac{1}{2}s$ and $r = -3\left(-\frac{1}{2}\right)$

$\therefore \; r = \frac{3}{2}$ and $s = -4$

EXERCISE 12G

1 $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -6 \\ r \\ s \end{pmatrix}$ are parallel. Find r and s.

2 Find scalars a and b given that $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} a \\ 2 \\ b \end{pmatrix}$ are parallel.

3 **a** Find a vector of length 1 unit which is parallel to $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$.

Hint: Let the vector be $k\mathbf{a}$.

b Find a vector of length 2 units which is parallel to $\mathbf{b} = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$.

H UNIT VECTORS

A **unit vector** is any vector which has a length of one unit.

For example:

- $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is a unit vector as its length is $\sqrt{1^2+0^2}=1$

$\mathbf{i}=\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{j}=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are 2-dimensional unit vectors in the positive x and y-directions respectively.

- $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$ is a unit vector as its length is $\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+0^2+\left(-\frac{1}{\sqrt{2}}\right)^2}=1$

$\mathbf{i}=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{j}=\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{k}=\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ are 3-dimensional unit vectors in the directions of the positive X, Y and Z-axes respectively.

EXERCISE 12H

1 Which of the following are unit vectors?

a $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ **b** $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ **c** $\begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$ **d** $\begin{pmatrix} -\frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$ **e** $\begin{pmatrix} \frac{2}{7} \\ -\frac{5}{7} \end{pmatrix}$

Notice that $\mathbf{v}=\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \Leftrightarrow \mathbf{v}=v_1\mathbf{i}+v_2\mathbf{j}+v_3\mathbf{k}$.

component form — unit vector form

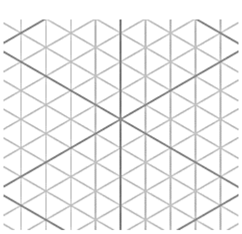
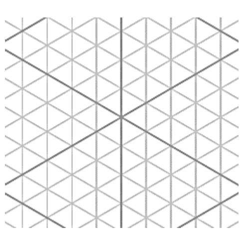
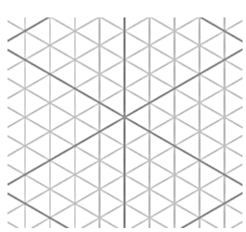
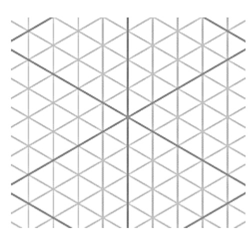
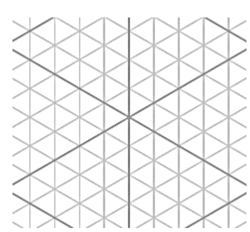
Thus, $\mathbf{v}=\begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}$ can be written as $\mathbf{v}=2\mathbf{i}+3\mathbf{j}-5\mathbf{k}$ and vice versa.

2 Write in terms of **i** and **j**:

a $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ **b** $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$ **c** $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ **d** $\begin{pmatrix} 0 \\ 7 \end{pmatrix}$ **e** $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

3 Write in component form; then graph the result

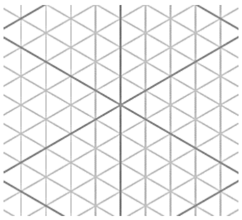
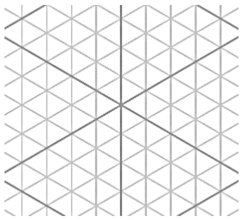
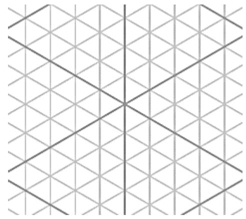
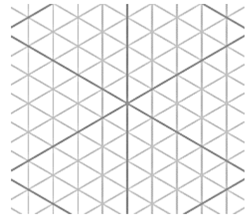
a $3\mathbf{i}+5\mathbf{j}$ **b** $5\mathbf{i}-4\mathbf{j}$ **c** $-4\mathbf{i}$ **d** $3\mathbf{j}$ **e** $\frac{\sqrt{3}}{2}\mathbf{i}-\frac{1}{2}\mathbf{j}$

Find the length of the 2-D vector $2\mathbf{i} - 5\mathbf{j}$.	As $2\mathbf{i} - 5\mathbf{j} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, its length is $\sqrt{2^2 + (-5)^2}$ $= \sqrt{29}$ units.

4 Express the following vectors in component form and find their lengths; then graph the result

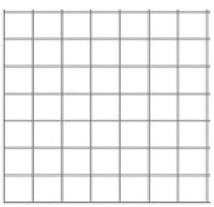
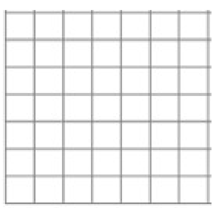
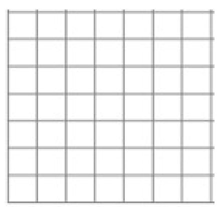
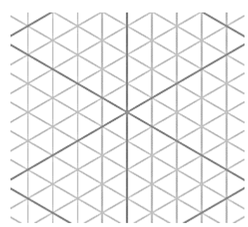
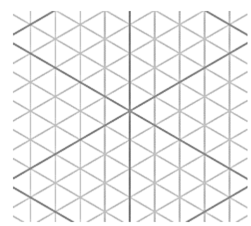
a $\mathbf{i} - \mathbf{j} + \mathbf{k}$ **b** $3\mathbf{i} - \mathbf{j} + \mathbf{k}$ **c** $\mathbf{i} - 5\mathbf{k}$ **d** $\frac{1}{2}(\mathbf{j} + \mathbf{k})$

Find k given that $\begin{pmatrix} -\frac{1}{3} \\ k \end{pmatrix}$ is a unit vector.	Since $\begin{pmatrix} -\frac{1}{3} \\ k \end{pmatrix}$ is a unit vector, $\sqrt{(-\frac{1}{3})^2 + k^2} = 1$ $\therefore \sqrt{\frac{1}{9} + k^2} = 1$ $\therefore \frac{1}{9} + k^2 = 1$ $\therefore k^2 = \frac{8}{9}$ $\therefore k = \pm\frac{\sqrt{8}}{3}$

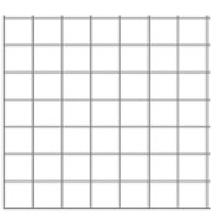
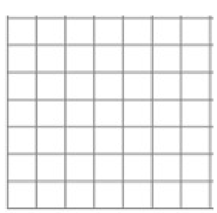
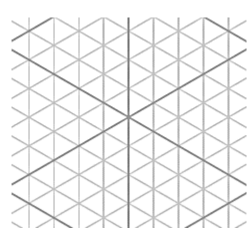
5 Find k for the unit vectors:

a $\begin{pmatrix} 0 \\ k \end{pmatrix}$ **b** $\begin{pmatrix} k \\ 0 \end{pmatrix}$ **c** $\begin{pmatrix} k \\ 1 \end{pmatrix}$ **d** $\begin{pmatrix} -\frac{1}{2} \\ k \\ \frac{1}{4} \end{pmatrix}$ **e** $\begin{pmatrix} k \\ \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$

- A unit vector in the direction of $\mathbf{v}$ is $\frac{1}{|\mathbf{v}|}\mathbf{v}$.
- A vector $\mathbf{b}$ of length k in the same direction as $\mathbf{a}$ is $\mathbf{b} = \frac{k}{|\mathbf{a}|}\mathbf{a}$.
- A vector $\mathbf{b}$ of length k which is *parallel to* $\mathbf{a}$ could be $\mathbf{b} = \pm\frac{k}{|\mathbf{a}|}\mathbf{a}$.

7 Find the unit vector in the direction of: **a** $\mathbf{i} + 2\mathbf{j}$ **b** $2\mathbf{i} - 3\mathbf{k}$ **c** $-2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$

Find a vector $\mathbf{b}$ of length 7 in the opposite direction to the vector $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

The unit vector in the direction of $\mathbf{a}$ is $\dfrac{1}{|\mathbf{a}|}\mathbf{a} = \dfrac{1}{\sqrt{4+1+1}}\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$$= \frac{1}{\sqrt{6}}\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}.$$

We now multiply this unit vector by -7. The negative reverses the direction and the 7 gives the required length.

Thus $\mathbf{b} = -\frac{7}{\sqrt{6}}\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$. Check that $|\mathbf{b}| = 7$.

8 Find a vector $\mathbf{b}$ in:

- **a** the same direction as $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and with length 3 units
- **b** the opposite direction to $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$ and with length 2 units
- **c** the same direction as $\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$ and with length 6 units
- **d** the opposite direction to $\begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix}$ and with length 5 units.

EXERCISE 12G

1 $r = 3,\ s = -9$ **2** $a = -6,\ b = -4$

3 **a** $\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{2}{3} \end{pmatrix}$ or $\begin{pmatrix} -\frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$ **b** $\begin{pmatrix} -\frac{4}{3} \\ -\frac{2}{3} \\ \frac{4}{3} \end{pmatrix}$ or $\begin{pmatrix} \frac{4}{3} \\ \frac{2}{3} \\ -\frac{4}{3} \end{pmatrix}$

EXERCISE 12H

1 **a** unit vector **b** unit vector **c** not a unit vector
d unit vector **e** not a unit vector

2 **a** $2\mathbf{i} - \mathbf{j}$ **b** $-3\mathbf{i} - 4\mathbf{j}$ **c** $-3\mathbf{i}$ **d** $7\mathbf{j}$ **e** $\frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}$

3 **a** $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ **b** $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$ **c** $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ **d** $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ **e** $\begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{pmatrix}$

4 **a** $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $\sqrt{3}$ units **b** $\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$ $\sqrt{11}$ units **c** $\begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}$ $\sqrt{26}$ units **d** $\begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$ $\frac{1}{\sqrt{2}}$ units

5 **a** $k = \pm 1$ **b** $k = \pm 1$ **c** $k = 0$
d $k = \pm\frac{\sqrt{11}}{4}$ **e** $k = \pm\frac{2}{3}$

7 **a** $\frac{1}{\sqrt{5}}(\mathbf{i} + 2\mathbf{j})$ **b** $\frac{1}{\sqrt{13}}(2\mathbf{i} - 3\mathbf{k})$ **c** $\frac{1}{\sqrt{33}}(-2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k})$

8 **a** $\frac{3}{\sqrt{5}}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ **b** $-\frac{2}{\sqrt{17}}\begin{pmatrix} -1 \\ -4 \end{pmatrix}$ **c** $\frac{6}{\sqrt{18}}\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$
d $-\frac{5}{3}\begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix}$