SL 1 **Name**_____

3D vectors in Component Form: WS #5 12E (332) #1-4,7,8,11

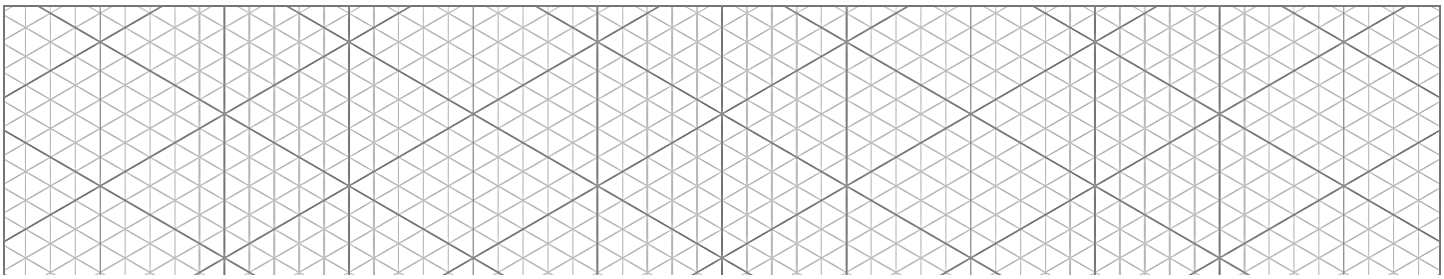
EXERCISE 12E

1 Consider the point T$(3, -1, 4)$.

a Draw a diagram to locate the position of T in space.

b Find $\overrightarrow{OT}$.

c How far is it from O to T?



If A(x_1, y_1, z_1) and B(x_2, y_2, z_2) are two points in space then:

$$\overrightarrow{AB} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix} \begin{matrix} \leftarrow x\text{-step} \\ \leftarrow y\text{-step} \\ \leftarrow z\text{-step} \end{matrix}$$

$\overrightarrow{AB}$ is called the 'vector AB' or the '**position vector of B relative to A**'.

If A is $(-1, 3, 2)$ and B is $(2, 1, -4)$, find:

a the position vector of A from B **b** the distance between A and B.

a The position vector of A from B is

$$\overrightarrow{BA} = \begin{pmatrix} -1-2 \\ 3-1 \\ 2-(-4) \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ 6 \end{pmatrix}$$

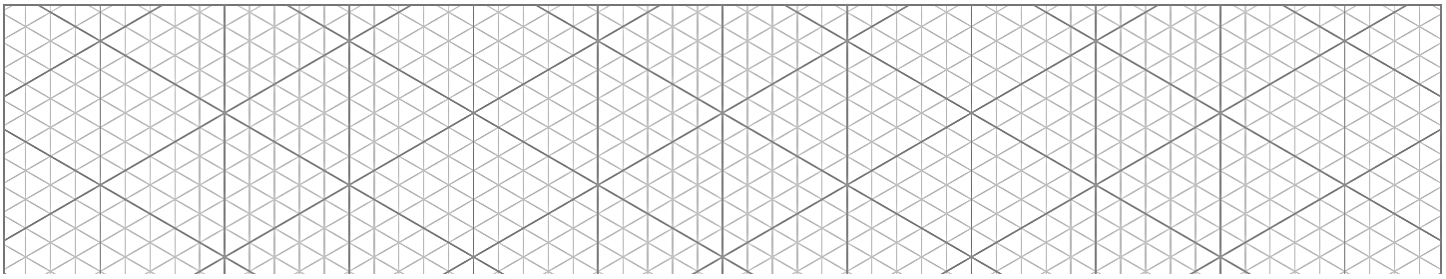
b $|\overrightarrow{AB}| = |\overrightarrow{BA}|$

$= \sqrt{9+4+36}$

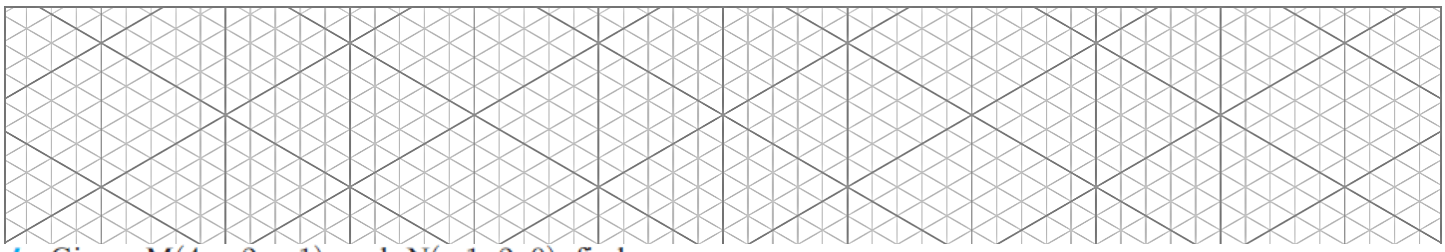
$= 7$ units

2 Given A$(-3, 1, 2)$ and B$(1, 0, -1)$ find:

a $\overrightarrow{AB}$ and $\overrightarrow{BA}$ **b** the lengths of $\overrightarrow{AB}$ and $\overrightarrow{BA}$.



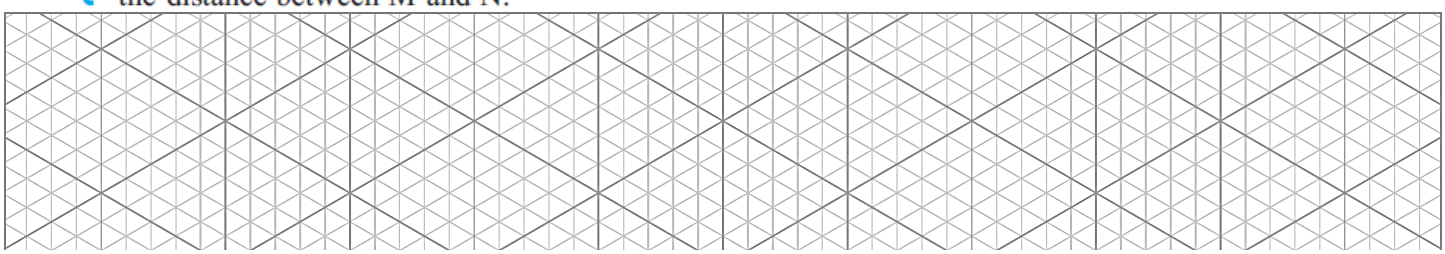
3 Given A$(3, 1, 0)$ and B$(-1, 1, 2)$ find $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{AB}$.



4 Given M$(4, -2, -1)$ and N$(-1, 2, 0)$ find:

a the position vector of M from N **b** the position vector of N from M

c the distance between M and N.



Find a, b, and c if $\begin{pmatrix} a-3 \\ b-2 \\ c-1 \end{pmatrix} = \begin{pmatrix} 1-a \\ -b \\ -3-c \end{pmatrix}$.

Equating components, $a-3=1-a$, $b-2=-b$ and $c-1=-3-c$

$\therefore \quad 2a=4, \quad 2b=2$ and $2c=-2$

$\therefore \quad a=2, \quad b=1$ and $c=-1$

7 Find a, b and c if: **a** $\begin{pmatrix} a-4 \\ b-3 \\ c+2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$ **b** $\begin{pmatrix} a-5 \\ b-2 \\ c+3 \end{pmatrix} = \begin{pmatrix} 3-a \\ 2-b \\ 5-c \end{pmatrix}$

8 Find scalars a, b and c if:

a $2\begin{pmatrix} 1 \\ 0 \\ 3a \end{pmatrix} = \begin{pmatrix} b \\ c-1 \\ 2 \end{pmatrix}$ **b** $\begin{pmatrix} 2 \\ a \\ 3 \end{pmatrix} = \begin{pmatrix} b \\ a^2 \\ a+b \end{pmatrix}$

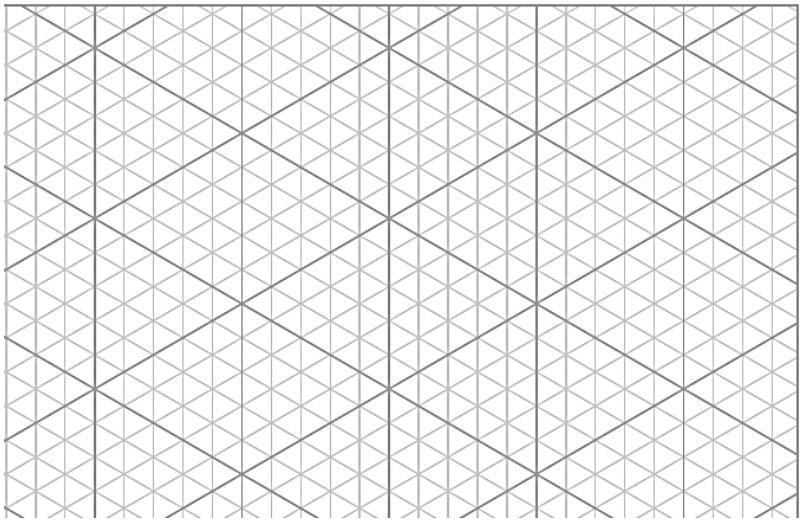
c $a\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + c\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix}$

11 PQRS is a parallelogram. P is $(-1, 2, 3)$, Q is $(1, -2, 5)$ and R is $(0, 4, -1)$.

a Use vectors to find the coordinates of S.

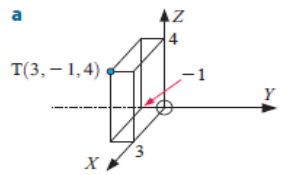
b Use midpoints of diagonals to check your answer.

C. Draw parallelogram PQRS



EXERCISE 12E

1 **a**



b $\overrightarrow{OT} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$ **c** OT $= \sqrt{26}$ units

2 **a** $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix}$, $\overrightarrow{BA} = \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$

b AB $= \sqrt{26}$ units BA $= \sqrt{26}$ units

3 $\overrightarrow{OA} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$, $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix}$

4 **a** $\overrightarrow{NM} = \begin{pmatrix} 5 \\ -4 \\ -1 \end{pmatrix}$ **b** $\overrightarrow{MN} = \begin{pmatrix} -5 \\ 4 \\ 1 \end{pmatrix}$ **c** MN $= \sqrt{42}$ units

7 **a** $a=5,\ b=6,\ c=-6$ **b** $a=4,\ b=2,\ c=1$

8 **a** $a=\frac{1}{3},\ b=2,\ c=1$ **b** $a=1,\ b=2$

c $a=1,\ b=-1,\ c=2$

11 **a** S$(-2, 8, -3)$