Unit 9 (DIFFERENTIAL EQUATIONS) REVIEW

1. Consider the differential equation $\frac{dy}{dx} = 3x^2$

   a) Find a solution y = *f*(x) to the differential equation satisfying $f(0) = 4$.
   b) Use the equation of tangent line of $f(0) = 4$ to approximate the value of $f(0.2)$
   c) Find the domain and range of the function y = *f*(x) found in part a.

2. Let f be a function with $f(2) = 1$ such that for all point (x,y) on the graph of f the slope is given by $\frac{2x^3+5}{3y}$.

   a) Find the slope of the graph of *f* at the point where x = 2.
   b) Write an equation for the line tangent to the graph of *f* at x = 2 and use it to approximate $f(2.1)$.
   c) Find *f*(x) by solving the separable differentiable equation $\frac{2x^3+5}{3y}$.
   d) Use your solution from part (c) to find $f(2.1)$.

3. Consider the differential equation given by $\frac{dy}{dx} = \frac{-xy}{3}$.

   a) Sketch a slope-field for the nine points.
      [(-1,1), (-1,2), (-1,3), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3)]

   b) Let $y = f(x)$ be the particular solution to the differential equation with $f(0) = 3$. Use the equation of the tangent line to approximate $f(0.2)$. Show all work.

   c) Find the particular solution $y = f(x)$ to the differential equation with $f(0) = 3$. Use your solution to find $f(0.2)$.

4. Given that the rate of growth of a population is proportional to the population at any given time, use the following information to answer the questions below. The initial population is 9.

   a) Use separation of variables to find the particular solution $y(x)$ to the differential equation with the given initial values.

   b) Given that the population grows to 72 after only 5 days, find the growth constant k.

   c) Use your calculator to find when the population will reach 200, if the growth continues at this same rate.

Answer Key

1. a) $y = x^3 + 4$ b) y (0.2) ≈ 4 c) D = all real R = all real

2. a) 7 b) f(2.1) ≈ 1.7 c) $y = \pm\sqrt{\frac{1}{3}x^4 + \frac{10}{3}x + C}$ d) f(2.1) = 1.575658

3. a) Slope Field b) y(0.2) ≈ 3 c) y(0.2) = 2.9800

4. a) $y = 9e^{kt}$ b) k = 0.41588 c) t = 7.4567 days