Unit 7 (Areas and Volumes) Review #2

Setup but do not solve the integrals for each. Do not use a calculator.

1. Use the curves $f(x) = x^2$ and $g(x) = 2x$ to answer the following.

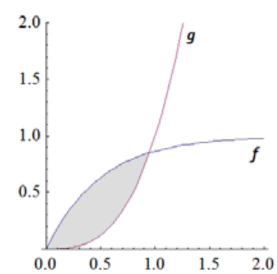
a) Find the area between the curves.
b) Find the average value of $f(x)$ over the interval [-1, 5].
c) Find the volume if the cross-sections are equilateral triangles perpendicular to the x-axis.
d) Find the volume of the solid rotated about the line $y = -3$.
e) Find the volume of the solid rotated about the line $x = -2$.

Solve using a graphing calculator.

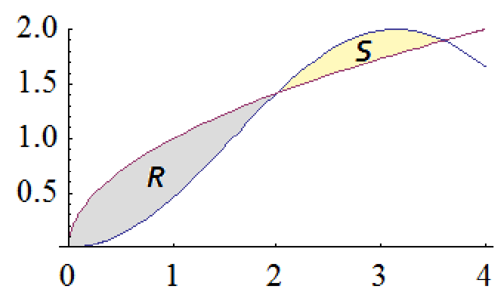
2. Find the volume of the solid of revolution generated by revolving the region bounded by $y = 2x^2$, $y = 0$, and $x = 2$ about:
a) the y–axis
b) the x–axis
c) the line $y = 8$

3. Find the volume of the solid of revolution generated by revolving the region bounded by $y = 6 - 2x - x^2$ and $y = x + 6$ about:
a) the x–axis
b) the line $y = 3$

4. The region in Quadrant I, bounded by the graph of $f(x) = 1 - e^{-x}$ and $g(x) = x^3$ is the base of a solid. Find the volume of this solid, if
a) each cross section perpendicular to the x-axis is an isosceles right triangle with one leg across the base of the solid.
b) each cross section perpendicular to the x-axis is a square.



5. Let R and S be the regions bounded by the graphs of $f(x) = 1 - \cos x$ and $g(x) = \sqrt{x}$ in Quadrant I.



a) Find the total area of the regions bounded by f and g in Quadrant I, that is, $R + S$
b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an equilateral triangle. Find the volume of this solid.
c) Region S is the base of another solid. For this solid, each cross section perpendicular to the x-axis is a semicircle. Find the volume of this solid.

6. Find the volume of the solid of revolution generated by revolving the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ about:

a) the x–axis
b) the y–axis
c) the line $x = 4$
d) the line $x = 6$

7. Given the equations $x = y^2$ and $x = -\frac{1}{2}y + 3$, find:

a) the area between the curves.
b) the average value of the function $y = -\sqrt{x}$ over the interval [1, 9].
c) the volume if the cross-sections are semi-circles perpendicular to the y-axis.
d) the volume rotated about the line $y = 2$.
e) the volume rotated about the y-axis.

Answers

1a) $A = \int_0^2 \left(2x - x^2\right) dx$

1b) $\frac{1}{6}\int_{-1}^{5} \left(x^2\right) dx$

1c) $V = \frac{\sqrt{3}}{4}\int_0^2 \left(2x - x^2\right)^2 dx$

1d) $V = \pi\int_0^2 \left[\left(2x+3\right)^2 - \left(x^2+3\right)^2\right] dx$

1e) $V = \pi\int_0^4 \left[\left(\sqrt{y}+2\right)^2 - \left(\frac{1}{2}y+2\right)^2\right] dy$

2a) 50.265
2b) 80.425
2c) 187.658
3a) 152.681
3b) 67.858
4a) 0.016
4b) 0.032
5a) 1.079
5b) 0.165
5c) 0.024
6a) 25.133
6b) 80.425
6c) 53.617
6d) 120.637
7a) 7.146
7b) -2.167
7c) 6.875
7d) 101.02
7e) 85.3