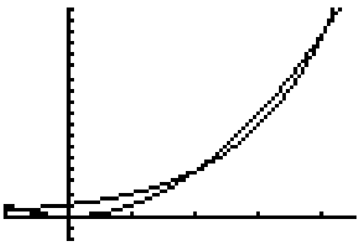
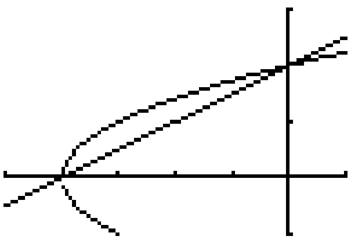
Area and Volume Review #1

Do your work on a separate sheet of paper. Use a graphing calculator when necessary.



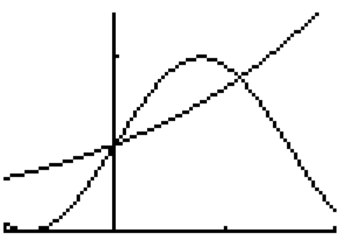
1. Let R_1 and R_2 be the regions enclosed by the graphs of f(x) = x² and $g(x) = 2^x$

a) Find the x- and y- coordinates of the three points of intersection of the graphs of *f* and *g*.
b) Without using absolute value, set up an expression involving one or more integrals that gives the total areas enclosed by the graphs of *f* and *g*. Do not evaluate.
c) Without using absolute value, set up an expression involving one or more integrals that gives volume of the solid generated by revolving R_1 about the line y = 4. Do not evaluate.



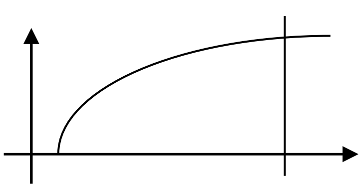
2. Consider the curve $y^2 = 4 + x$ and the chord AB joining points A (-4, 0) and B (0, 2) on the curve.

a) Find the x- and y- coordinates of the point on the curve where the tangent line is parallel to chord AB.
b) Find the area of the region enclosed by the curve and chord AB.
c) Find the volume of the solid generated when the region, *R*, defined in part (b) is revolved about the x-axis.

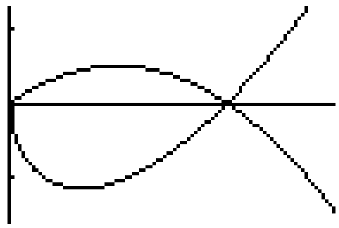


3. Let *f* and *g* be the functions given by f(x) = 1 + sin(2x) and $g(x) = e^{\frac{x}{2}}$. Let *R* be the shaded region in the first quadrant enclosed by the graphs of *f* and *g* as shown to the left.

a) Find the area of *R*.
b) Find the volume of the solid generated when *R* is revolved about the x-axis.
c) The region *R* is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from y = f(x) to y = g(x). Find the volume of this solid.

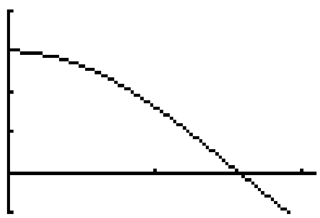


4. Let *R* be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line x = 10, and the x-axis.
a) Find the area of *R*.
b) Find the volume of the solid generated when *R* is revolved about the horizontal line y = 3.
c) Find the volume of the solid generated when *R* is revolved about the vertical line x = 10.



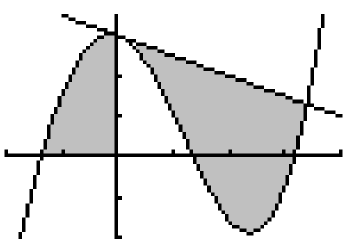
5. Let *f* and *g* be the functions given by f(x)=2x(1 – x) and $g(x)=3(x-1)\sqrt{x}$ for $0\le x\le 1$. The graphs of *f* and *g* are shown to the left.

a) Find the area of the shaded region enclosed by the graphs of *f* and *g*.
b) Find the volume of the solid generated when the shaded region enclosed by the graphs of *f* and *g* is revolved about the horizontal line y = 2.
c) Let *h* be the function given by *h(x) = kx(1 – x) for* $0\le x\le 1$. For each *k*>0, the region (not shown) enclosed by the graphs of *h* and *g* is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of *k* for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of *k*.



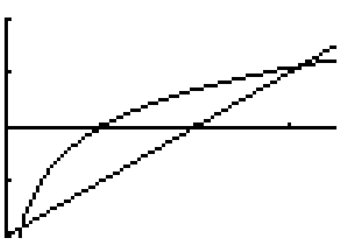
6. Let *f* be the function given by *f(x) = 3 cos x*. As shown to the left, the graph of *f* crosses the y-axis at point *P* and x-axis at point *Q*.

a) Write an equation for the line passing through points *P* and *Q*.
b) Write the equation for the line tangent to the graph of *f* at point *Q*. Show your analysis.
c) Find the x-coordinate of the point on the graph of *f*, between points *P* and *Q*, at which point the line tangent to the graph of *f* is parallel to the line *PQ*.
d) Let *R* be the region in the first quadrant bounded by the graph of *f* and the line segment *PQ*. Write an integral expression for the volume of the solid generated by revolving *R* about the x-axis. DO NOT EVALUATE.



7) Let *f* be the function given by $f(x)=\frac{x^3}{4}-\frac{x^2}{3}-\frac{x}{2}+3\cos x$. Let *R* be the shaded region in the second quadrant bounded by the graph of *f*, and let *S* be the shaded region bounded by the graph of *f* and line *l*, the line tangent to the graph of *f* at x = 0, as shown to the left.

a) Find the area of *R*.
b) Find the volume of the solid generated when *R* is rotated about the horizontal line y = -2.
c) Write, but do not evaluate, an integral expression that can be used to find the area of *S*.



8) Let *R* be the shaded region bounded by the graph of y = ln x and the line y = x – 2, as shown to the left.

a) Find the area of *R*.
b) Find the volume of the solid generated when *R* is rotated about the horizontal line y = -3.
c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when *R* is rotated about the y-axis.

Answers for Review Worksheet

1) a) (-0.767, 0.588) (2, 4) and (4, 16)

   b) $\int_{-0.767}^{2}(2^x - x^2)dx + \int_{2}^{4}(x^2 - 2^x)dx$

   c) $\pi\int_{-0.767}^{2}[(4-x^2)^2 - (4-2^x)^2]dx$

2) a) (-3, 1)

   b) 4/3

   c) 8π/3

3) a) 0.429

   b) 4.267

   c) 0.0777

4) a) 18

   b) 212. 057 or 212.058

   c) 407.150

5) a) 1.133

   b) 16.179

   c) $\int_{0}^{1}(kx(1-x) - 3(x-1)\sqrt{x})^2 dx = 15$

6) a) $y = \frac{-6}{\pi}x + 3$

   b) $y = -3x + \frac{3\pi}{2}$

   c) 0.690

   d) $\pi\int_{0}^{\frac{\pi}{2}}[9\cos^2 x - (\frac{-6}{\pi}x + 3)^2]dx$

7) a) 2.903

   b) 59.361

   c) $\int_{0}^{3.38987}[(3 - \frac{1}{2}x) - f(x)]dx$

8) a) 1.949

   b) 34.198 or 34.199

   c) $\pi\int_{-1.841}^{1.146}[(y+2)^2 - (e^y)^2]dy$